Decision Making Using Hurwicz Criteria

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Abstract: - The purpose of this paper is to study and discuss the effect of the different types of two, three and four wheeler vehicles and operating their features like time, distance and satisfaction according to their speed performance. In this paper, we deal with multi-criteria decision making problem under uncertainty fuzzy logic and fuzzy set. Defuzzification is the process of producing a quantifiable result in Crisp logic, given fuzzy sets and corresponding membership degrees. In this paper, decision making is applied using Hurwicz criteria and defuzzification is done using area of region for triangular, trapezoidal and octagonal fuzzy numbers. Hurwicz criteria are mostly used for decision making if the data are precise and quantitative. On considering some numerical examples, both methods are applied to the defuzzified groups of information or data to find the best action or performance of vehicles.

Keywords: Defuzzification, Multi criteria decision making, Hurwicz criteria, triangular, trapezoidal and Octogonal fuzzy number.

AMS Subject Classification : 90C70

1. Introduction

Decision making are mostly used in our daily life in many situations [1, 15]. Decision making are forced to rely on their own subjective ideas of the efficiency of possible alternatives and importance of diverse criteria. The Decision making process start with the intelligence phase, where the relative or decision maker has examined the problem and it is identified, statement is defined and analyses decision maker’s attitude in the fuzzy part network. Recent research has recognized that multi-criteria decision making (MCDM) [16, 19, 2, 4] should take account of uncertainty, risk and confidence. To illustrate the computation process and demonstrate the feasibility of the results we use a travel problem that has been used previously to assess MCDM.

Defuzzification is a conversion of fuzzy set into crisp set or values. Norman Fenton and Wei Wang [13] proposed a useful method to tackle this type of decision making problems under uncertainty. They analyses the risk and confidence attitude of a decision maker in multi criteria decision making for triangular fuzzy numbers and also extend to trapezoidal and octagonal fuzzy numbers. There are many different types of methods of defuzzification available for trapezoidal fuzzy numbers [10, 11]. Broekhoven and Baets [3] used three methods such as discretisation, modified
transformation function and slope-based method to perform the center of gravity (COG) defuzzification. The center of gravity method and the mean of maxima method are mostly used in fuzzy mathematics.

The area method for triangular, trapezoidal and octagonal fuzzy number exhibit the property of continuity that make them suitable for fuzzy controller. Malini and Kennedy [12] defined octagonal fuzzy numbers and their arithmetic operations.

The Hurwicz Criterion firstly presented in a paper 1951, is probably the earliest novel contribution to the field of economics for which Leo [7] has been recognized. It provides a formula for balancing pessimism and optimism in decision making under uncertainty that is when future conditions are to some extent unknown. This method is also known as decision making criterion. A defining feature of the Hurwicz Criterion is that it allows the decision makers to simultaneously take into account both the best and the worst possible outcomes. To do this for each given action the decision makers chooses a “coefficient of pessimism”, and “coefficient of optimism” called alpha (α) and (1 - α), where α, (1-α) are maximum pay off and minimum payoff and calculate the balance weighted outcome in uncertain situations. And find an even distribution of weights between the two absolute optimism and pessimism. And α is a decimal number between 0 and 1, this number determined the emphasis on the worst possible outcomes. Then the number (α) determines the emphasis to be placed on the best outcomes. So, if the coefficient of pessimism is 0.8, then the emphasis on the best outcomes will be 0.2. And α = 0.5 show that DM is neither optimistic nor pessimistic [8, 18]. In this article, using Hurwicz criteria for triangular fuzzy numbers we consider only the risk attitude of the decision maker and same applied for trapezoidal and octagonal fuzzy numbers. The fuzzy numbers are defuzzified using area of region (AOR) and Hurwicz criteria is applied to the defuzzified data [9].

2. Definition by Area of Region and Decision Making


If X is a universe of discourse and x is a particular element of X, then a fuzzy set a defined on X and can be written as a collection of ordered pairs

$$A= \{(x, \mu(x)), x \in X\}$$

where $\mu(x)$ is a membership function.

Definition 2.2. [7, 18] Hurwicz Criteria:-

The Hurwicz criteria joined a weighted value from the maximum payoff and the Minimum payoff of the strategies. Basically Hurwicz criteria are a combination of optimistic and pessimistic decision, where optimistic (1 - α) is also known as Maximin and pessimistic (α) is also known as
Maximax. First select coefficient of realism $\alpha$, with a value between 0 and 1. When $\alpha$ is close to 1, decision maker is optimistic about future and when $\alpha$ is close to 0, decision maker is pessimistic about future.

**Weighted Outcome**

$$\text{Weighted Outcome} = \alpha (\text{maximum payoff}) + (1 - \alpha) (\text{minimum payoff})$$

**Definition 2.3.** [5, 19] **Triangular Fuzzy Number:**

A fuzzy number $A = (a, b, c)$ is called Triangular Fuzzy Number if its membership function $\mu(x)$ is given by

$$\mu_A(x) = \begin{cases} 
0, & x \leq a \\
\frac{x-a}{b-a}, & a < x \leq b \\
1, & x = b \\
\frac{c-x}{c-b}, & b < x \leq c \\
0, & x \geq c.
\end{cases}$$

This number is based on three-value judgment $a$, $b$, and $c$, where $a$, $b$ and $c$ is the lower, most and upper possible value.

**Definition 2.4.** [5, 22, 23] **Trapezoidal Fuzzy Number:**

We can define Trapezoidal Fuzzy Number $\tilde{A}$ as $(a, b, c, d)$, the membership function $\mu(x)$ of this fuzzy number will be interpreted as follows

$$\mu_A(x) = \begin{cases} 
0, & x \leq a \\
\frac{x-a}{b-a}, & a < x \leq b \\
1, & b < x \leq c \\
\frac{c-x}{d-c}, & c < x \leq d \\
0, & x \geq d
\end{cases}$$

**Definition 2.5.** [3, 12] **Octagonal Fuzzy Number:**

A fuzzy number $A$ denoted by $(a, b, c, d, e, f, g, h)$ is a normal octagonal fuzzy number whose membership function $\mu(x)$, where $a$, $b$, $c$, $d$, $e$, $f$, $g$, $h$ are real numbers is given as
Definition 2.6. [16] Fuzzy MCDM approach:-

A fuzzy MCDM for a general multi criteria decision problem with m alternatives $\bar{A}_i \ (i = 1, 2, 3, \ldots, m)$ and n criteria $C_j \ (j = 1, 2, \ldots, n)$ can be expressed as $D = (x_{ij} \ \text{where} \ i = 1, 2, \ldots, m \ \text{and} \ j = 1, 2, \ldots, n \ \text{where} \ D \ \text{is decision matrix (where the entry} \ x_{ij} \ \text{represents the rating of alternative} \ A_i \ \text{with respect to criteria} \ C_j). \ \text{Basically criteria may be classified in two ways as Benefit criteria or Cost criteria. Benefit criteria represent higher the value} \ x_i \ \text{of better for the DM and Cost criteria represent lower the value of} \ x_i \ \text{better for DM. Since, it was proposed to consider fuzzy, as opposed to crisp values in} \ D \ \text{and} \ W, \ \text{the following notation} \ D[x_{ij}] \ \text{and weight vector of DM is represent as} \ W = (w_j) = (w_1, w_2, w_3, \ldots, w_m) \ \text{satisfied} \ \sum_{j=1}^{m} w_j = 1, \ \text{where} \ w_j \ \text{represents the fuzzy weight of criterion} \ C_j.$

Definition 2.7. [14, 20] Normalization:-

The fuzzy numbers in the decision matrix are normalized as the Performance matrix: $\hat{P} = [p_{ij}]$. This process is applied to deal with criteria of different scales. It’s denoted as $(P_{ij})$, where

\[
P_{ij} = (\frac{x_{ij1}}{M}, \frac{x_{ij2}}{M}, \frac{x_{ij3}}{M}), \quad M = \max (x_{ij3}) \ \text{for benefit criteria}
\]

or

\[
P_{ij} = (\frac{N-x_{ij1}}{N}, \frac{N-x_{ij2}}{N}, \frac{N-x_{ij3}}{N}), \quad N = \max (x_{ij3}) \ \text{for cost criteria}.
\]

This method preserves ranges of normalized triangular fuzzy numbers to $[0, 1]$.  

Definition 2.8. [14, 20] Weighting the criteria. 

The multiplication of weight vector with decision matrix represent the weighted performance matrix and it’s denoted by $\hat{P}$ written as $\hat{P} = [p_{ij}]$, where

$P_{ij1} = w_{j1} \times P_{i1j}, \quad P_{ij2} = w_{j2} \times P_{i2j} \ \text{and} \ P_{ij3} = w_{j3} \times P_{i3j}, \ \text{where} \ i=1, 2, 3, \ldots, m \ \text{and} \ j=1, 2, \ldots, n.$
3. Procedure

The algorithm was carried out by the following steps:-
1) Express the problem in the form of fuzzy decision matrix.
2) Find the performance matrix by normalizing each element of the matrix.
3) Obtain the weighted performance matrix by multiplying the performance matrix and its corresponding weight.
4) Consider each triangular fuzzy number as the vertex of a triangle in the form (a, 0), (b, 1), (c, 0) and find the area of each triangle. (Same procedure I adopted for the trapezoidal and octagonal fuzzy numbers).
5) Apply Hurwicz criteria to find the weighted outcome for each row.
6) The maximum value of the entire weighted outcome will be the best action.


Table 1 shows the decision matrix and its corresponding weight for the different types of vehicle feature problem. In this example, the different types of vehicle judge time and distance according to maximum speed of vehicles are measured in unit minute and meter respectively. The criterion comfort is a value criterion measured on a scale from 1 to 10. Let us consider the ratings in the decision matrix expressed as triangular fuzzy numbers (for example, two wheeler like bike journey to college most typically time 31 minutes but it can be as low as 29 minutes and as high as 33 minutes ). [11] The problem was illustrated with 3 different assumptions. The ratings in the decisions matrix were expressed as A) triangular, B) trapezoidal and C) octagonal fuzzy numbers:

A. [21] Triangular Fuzzy Numbers: - Table 1: Decision Matrix

<table>
<thead>
<tr>
<th>Types of vehicles</th>
<th>Maximum speed of vehicles</th>
<th>Time of vehicles ( \omega = 0.3 )</th>
<th>Distance of vehicles ( \omega = 0.5 )</th>
<th>Satisfy ( \omega = 0.2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Wheeler</td>
<td>(25, 35, 45)</td>
<td>(20, 31, 33)</td>
<td>(145.67, 300.72, 326.17)</td>
<td>(5, 6, 7)</td>
</tr>
<tr>
<td>Three Wheeler</td>
<td>(20, 30, 40)</td>
<td>(30, 40, 55)</td>
<td>(102.50, 190.68, 330.80)</td>
<td>(6, 8, 9)</td>
</tr>
<tr>
<td>Four Wheeler</td>
<td>(60, 70, 80)</td>
<td>(32, 43, 55)</td>
<td>(498.72, 723.01, 910.50)</td>
<td>(2, 5, 7)</td>
</tr>
</tbody>
</table>

Normalized fuzzy decision matrix was constructed and shown in Table 2

Table 2: Performance matrix
Types of vehicles | Time of vehicles $\omega = 0.3$ | Distance of vehicles $\omega = 0.5$ | Satisfy $\omega = 0.2$
--- | --- | --- | ---
Two Wheeler | (0.400, 0.436, 0.636) | (0.641, 0.669, 0.840) | (0.222, 0.333, 0.444)
Three Wheeler | (0, 0.272, 0.454 ) | (0.636, 0.790, 0.887) | (0, 0.111, 0.333)
Four Wheeler | (0.036, .0.218, 0.418) | (0, 0.205, 0.457 ) | (0.222,0.444, 0.777)

The weighted performance matrix was constructed by multiplying the performance matrix with its corresponding weight as

$$A= \begin{bmatrix}
(0.120,0.130,0.190) & (0.320, 0.334,0.420 ) & ( 0.444, 0.666, 0.888 ) \\
(0 , 0.0816 , 0.136) & (0318,0.395, 0.443) & (0, 0.022,0.066) \\
(0.0108,0.0654,0.125) & ( 0, 0.102,0.228 ) & ( 0.044,0.088,0.1554) \\
\end{bmatrix}.$$  

Consider each triangular fuzzy number as the vertex of a triangle in the form (a, 0), (b, 1), (c, 0). The triangular fuzzy decision matrix can be written as

$$A= \begin{bmatrix}
(0.120,0.130,0.190) & (0.320, 0.334,0.420 ) & ( 0.444, 0.666, 0.888 ) \\
90,0.0816 , 0.136) & (0318,0.395, 0.443) & (0, 0.022,0.066) \\
(0.0108,0.0654,0.125) & ( 0, 0.102,0.228 ) & ( 0.044,0.088,0.1554) \\
\end{bmatrix}.$$  

The area of a triangle is defined as the total space that is enclosed by any particular triangle. The basic formula to find the area of a given triangle is $A = \frac{1}{2} \times b \times h$, where $b$ is the base and $h$ is the height of the given triangle.

**(Area of triangle)** $A = \frac{1}{2} \times \text{base} \times \text{height},$

$A = \frac{1}{2} \times (c-a)$, since (h=1)

Each triplet represents a triangle. Table 3 was obtained by area of triangle (fig-1)

**Table 3: Defuzzified matrix**

| Types of vehicles | Time of vehicles $\omega = 0.3$ | Distance of vehicles $\omega = 0.5$ | Satisfy $\omega = 0.2$
--- | --- | --- | ---
Two Wheeler | 0.035 | 0.050 | 0.022
Three Wheeler | 0.068 | 0.062 | 0.033
Four Wheeler | 0.0575 | 0.114 | 0.055

**Table 4: Maximum and Minimum Payoffs**
<table>
<thead>
<tr>
<th>Types of vehicles</th>
<th>Time of vehicles $\omega = 0.3$</th>
<th>Distance of vehicles $\omega = 0.5$</th>
<th>Satisfy $\omega = 0.2$</th>
<th>Row Maximum</th>
<th>Row Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Wheeler</td>
<td>0.035</td>
<td>0.050</td>
<td>0.022</td>
<td>0.050</td>
<td>0.022</td>
</tr>
<tr>
<td>Three Wheeler</td>
<td>0.068</td>
<td>0.062</td>
<td>0.033</td>
<td>0.068</td>
<td>0.033</td>
</tr>
<tr>
<td>Four Wheeler</td>
<td>0.0575</td>
<td>0.114</td>
<td>0.055</td>
<td>0.114</td>
<td>0.055</td>
</tr>
</tbody>
</table>

Hurwicz criteria were used to find the weighted outcome in Table 4.

Weighted outcome = $\alpha$ (maximum payoff) + (1 - $\alpha$) (minimum payoff).

Table 5: Weighted Outcome, for different values of $\alpha$

<table>
<thead>
<tr>
<th>Types of vehicles</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Wheeler</td>
<td>0.022</td>
<td>0.0248</td>
<td>0.0276</td>
<td>0.0304</td>
<td>0.0332</td>
<td>0.036</td>
<td>0.0388</td>
<td>0.046</td>
<td>0.044</td>
<td>0.047</td>
<td>0.05</td>
</tr>
<tr>
<td>Three Wheeler</td>
<td>0.033</td>
<td>0.0365</td>
<td>0.0400</td>
<td>0.0435</td>
<td>0.047</td>
<td>0.054</td>
<td>0.0575</td>
<td>0.060</td>
<td>0.064</td>
<td>0.067</td>
<td>0.068</td>
</tr>
<tr>
<td>Four Wheeler</td>
<td>0.055</td>
<td>0.609</td>
<td>0.0668</td>
<td>0.0727</td>
<td>0.0786</td>
<td>0.084</td>
<td>0.0904</td>
<td>0.096</td>
<td>0.102</td>
<td>0.108</td>
<td>0.114</td>
</tr>
</tbody>
</table>

Defuzzification by area of region and decision making

Table 6: Ranking Order, for different values of $\alpha$

<table>
<thead>
<tr>
<th>Types of vehicles</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
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<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Wheeler</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
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<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Three Wheeler</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
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<td>2</td>
</tr>
<tr>
<td>Four Wheeler</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6 gives the ranking order under different risk attitude of the decision maker. The ranking order suggests that any decision maker with fair or extreme optimistic approach will choose the
car as first alternative and the three wheelers like as auto the second alternative and third alternative two wheelers like as bike.

**B) [23] Trapezoidal fuzzy numbers:-**

The ratings in the decision matrix are assumed as trapezoidal fuzzy numbers and same method apply in this case.

**Table 7: Decision Matrix**

<table>
<thead>
<tr>
<th>Types of vehicles</th>
<th>Maximum speed of vehicles</th>
<th>Time of vehicles $\omega = 0.3$</th>
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<tr>
<td>Three Wheeler</td>
<td>(20, 30, 35, 40)</td>
<td>(30, 40, 47, 55)</td>
<td>(102.50, 190.68, 238.59, 330.80)</td>
<td>(6, 7, 8, 9)</td>
</tr>
<tr>
<td>Four Wheeler</td>
<td>(60, 70, 75, 80)</td>
<td>(32, 43, 50, 55)</td>
<td>(498.72, 723.01, 860.20, 910.50)</td>
<td>(2, 5, 6, 7)</td>
</tr>
</tbody>
</table>

Normalized weighted performance and defuzzified matrix were found using area of trapezium

\[
A = \frac{1}{2} (a + b) h
\]

where, \( h \) = height

\((\text{Area of trapezium}) \ A = \frac{1}{2} \times [(d-a) + (c-d)] \times \text{height},\)

\[
\text{Area of Trapezium} = \frac{1}{2} \times [(d-a) + (c-d)] \times h
\]

\[
\text{Area of Trapezium} = \frac{1}{2} \times [(d-a) + (c-d)], \text{ (since } h=1)\]

The same procedure was adopted to find the ranking order and shown in Table 8.

**Table 8: Ranking Order, for different values of } \alpha \]

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http://www.webology.org
Table 8 gives the same ranking order for trapezoidal fuzzy number.

C) Octagonal Fuzzy Numbers:

The ratings in the decision matrix are assumed as octagonal fuzzy numbers

<table>
<thead>
<tr>
<th>Types of vehicles</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
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</thead>
<tbody>
<tr>
<td>Two Wheeler</td>
<td>3</td>
<td>3</td>
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<tr>
<td>Three Wheeler</td>
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<tr>
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</tr>
</tbody>
</table>

Normalized weighted performance and defuzzified matrix were found using area of trapezium (fig-3)

\[
\text{Area of Region} = \text{Area of Trapezium ABCD} + \text{Area of Trapezium EFGH}
\]

Area of Region = \(\frac{1}{2} \times h_1 \times [(h - a) + (g - b)] + \frac{1}{2} \times h_2 \times [(f - c) + (e - d)]\), (since \(h_1, h_2 = 0.5\))

The same procedure was adopted to find the ranking order and shown in Table 10
Table 10: Ranking Order, for different values of α

<table>
<thead>
<tr>
<th>Types of vehicles</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Wheeler</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
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<td>3</td>
</tr>
<tr>
<td>Three Wheeler</td>
<td>2</td>
<td>2</td>
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<td>2</td>
<td>2</td>
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<td>2</td>
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<td>2</td>
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<td>2</td>
</tr>
<tr>
<td>Four Wheeler</td>
<td>1</td>
<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
</tr>
</tbody>
</table>

Table 10 gives the ranking order for octagonal fuzzy numbers with same input. It was noted that the ranking order for triangular, trapezoidal and octagonal fuzzy numbers were same for this problem.

5. Conclusion.

In this paper we worked on a different type of vehicles performance problem where we have some fuzzy outputs which need to defuzzify to give a crisp result. We have presented defuzzification techniques by area of region (AOR) and also presented Hurwicz criteria method to find the ranking order and suggests that any decision maker with fair or extreme optimistic approach will choose the car as first alternative and the three wheelers like as auto, the second alternative and third alternative two wheelers like as bike. As per experimental observations, both methods has given approximately same output for the triangular fuzzy numbers. Similarly same output for the trapezoidal and the octagonal fuzzy numbers using input value were same for this problem. In this paper region (AOR) and Hurwicz criteria defuzzification method are very useful for multi criteria decision making problem to find the best action under different risk Attitudes of the decision Maker.

References


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