Webology (ISSN: 1735-188X)
Volume 19, Number 2, January, 2022

Semiotics in Integers: How Can the Semiosis Connections Occur in Problem Solving?

Hersiyati Palayukan[1], Purwanto[2], Subanji[3*], Sisworo[4]
Faculty of Mathematics and Science - Universitas Negeri Malang, Indonesia
hersiyati.palayukan.1703119@students.um.ac.id[1], purwanto.fmipa@um.ac.id[2],
subanji.fmipa@um.ac.id[3], sisworo.fmipa@um.ac.id[4]
*Corresponding Author: subanji.fmipa@um.ac.id[3]

Abstract

Semiotics has a role to show the importance of the meaning of signs in mathematics learning. Integer is part of mathematics whose conceptual understanding is highly dependent on the interpretation of signs, especially in the problem-solving process that involves the cognitive process. This paper aims to describe students' cognitive processes in problem solving, through the relationship between Peirce's triadic components (semiosis), namely representamen (R) - object (O) - interpretant (I). This research is a descriptive qualitative research through the stages of giving questions (problems), determining the subject, interviews, data analysis, and conclusions. Subjects are selected based on consistency in problem solving using iconic representamen and symbolic representamen. The results of this study found four stages in the process of semiosis connection:1)
Observing occurs when the subject first understands and produces an object in his mind, 2) creating/producing, namely when the subject manifests the object in his mindinto a new represent a men, 3) confirming occurs when the subject searches for truth byconnecting the representation with a correspondingobject, and 4) reflecting occurs when the subject checks by connecting all represent a men with the object to obtain the final interpretation.

Keyword. Semiotics, semiosis, signs, integers, problem solving

Introduction

Semiotics is the study of rules or systems that allow signs to have meaning(Kriyantono, 2007: 261). The semiotic tradition consists of sign theories on how a signs represents an ideas, circumstances, objects, feelings, conditions or situations outside the signs themselves (Littlejohn, 2009: 53). Therefore, semiotics aims to determine the meanings contained in a sign. According to Charles Sanders Peirce's theory, logic underlies semiotic theory, because logic explores how people reason, whereas according to Peirce reasoning is done through the help of signs. Signs allow us to think, relate to other people and give meaning to what the universe displays. Peirce's semiotic theory defines the sign model as trichotomic or triadic, and has no structural features at all (Hoed, 2002: 21). The basic principle is that a sign is representative, that is, a sign is something that stands for something else (something that represents something else), Peirce views that a sign is an object that represents something (thought) to another (Peirce, 1991). Signs are all things, both physical and mental, that are given meaning by humans. Everything that is meant could be in the form of ideas, thoughts, experiences (something gone through), or feelings, and it is not only limited to objects. So a sign is a sign only if it has meaning (Hoed, 2014: 5). For Peirce, signs and their meanings are a cognitive process called semiosis.

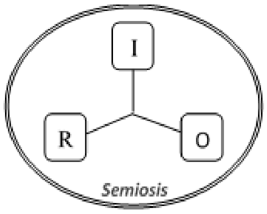

Figure 1. Peirce sign triadic model; R (representamen) - O (object) - I (interpretant)

Semiosis (semiotic process) is the process of meaning and interpretation of signs. The process of meaning occurs in the relationship between "reality" and "what is in human cognition" (Hoed, 2014). According to Peirce's philosophy, semiosis (meaning of signs) is a relationship between triadic/ trichotomy between signs or representamen (R) - object (O) - interpretant (I). R is the part of the sign that can be perceived physically or mentally, which refers to something that is represented by it (O). Then I is part of the process that interprets the relationship between R and O. So semiosis is the sign formation process that starts from representamen which is spontaneously related to object in human cognition and then given a certain interpretation by the human concerned as interpretant. Semiosis is a process of constructing the meaning of a sign and is a process that is closely related to mathematics education. In this case the relationship (significance) between semiosis and mathematics lies in the sign and its usage in every branch of mathematics. Objects in mathematics require sign vehicles as a representation to represent these objects so that they can be understood or interpreted. Therefore, as a strong theoretical lens, semiotics has great potential to investigate various research topics in the field of mathematics education (Presmeg, 2016). Because semiosis itself is a cognitive process, it surely cannot be separated from the meaning of concepts in solving mathematical problems which also involve processing cognition.

Researches related to semiotics in various mathematical concepts have received special attention. These studies show the role of semiotics in the studying and learning process as well as in problem solving. Semiotics is seen as a tool to describe mathematical activities in the learning and teaching of several concepts, such as the analysis of semiotic systems involved in basic arithmetic concepts (Godino et al. 2011), semiotic structures with respect to geometry (Dimmel& Herbst, 2015), cognitive activity analysis on proving a function of absolute values (Pino et al. 2017). These studies have proven that learning and teaching mathematics related to cognitive activity is a semiotic activity.

More specifically, research on semiotics using Peirce's semiotic sign principles and involves semiosis components such as Miller (2015), found that signs in learning help students determine how to identify and calculate growth patterns. On the other hand, triadic components serve as a tool in revealing students' conceptual understanding of the signs on the function graph (Mudaly, 2014). In addition, a semiotic analysis on learning geometric concepts Daher (2014), shows that students are involved in semiotic activities, such as manipulative, discussion and problem situations. Still on the same topic, Alshwaik (2010) developed a semiotic analysis framework as a tool to analyze the types of meaning of geometric diagrams in mathematical discourse.Likewise, through a semiotic process, Palayukan et al. (2020) found that the construction of students' understanding of geometry was determined by the semiotic structure contained in the geometry diagram.

Mathematics learning is a multimodal discourse that consists of various representations and communications, such as gestures, visual forms, language and algebraic notation(Morgan, 2014) both between teachers and students or between fellow students (Zayyadi et al. 2020). These different modes have and carry different potential meanings. Signs play a role in making mathematics look real (Radford, 2003). Signs also help students develop mathematical understanding (Sabena, et al. 2005). As Torigoe& Gladding (2011) found that student performance is highly correlated with understanding of symbols. Likewise, according to Quinnel& Carter (2012) in students thinking mathematically, mathematical ideas which are communicated in writing can only be achieved only by the presence of symbols. Therefore, in order to be able to work with symbols, it is not enough for one to recognize them in the text, but also to choose the correct interpretation and context-based. The same symbol can contain different meanings in different contexts, resulting in different interpretations for students.

In relation to education, especially mathematics, signs such as words, symbols, diagrams, graphics and schemes are mediators that connect external reality and internal mental processes called semiotic mediation processes (Mudaly 2014). The mental process here is the development of the social interaction concept which then experiencing internalization through semiosis (Vile, 1993). Berger (2006) stated that the use of mathematical signs is the first step in the conceptual process that occurs before deriving mathematical meaning. The semiotic approach to mathematics education introduces the concept of "semiotic systems" as a means of describing mathematical activities. The semiotic system is formed by a series of signs, the rules of sign production, and the underlying structure of meaning.

Processing and interpretation of signs cannot be separated from mathematical activities, either in meaning construction or in problem solving. According to Santi (2011), mathematics activities and mathematics learning are intrinsically parts of semiotic activities. She argued that without turning to the transformation of signs in the semiotic system, mathematics will not develop into the subtle form of rationality we know today. Even Godino (2011) found that the semiotic system is a tool for describing properties of mathematical objects. Thus we can say that the concepts found in Peirce's semiotics are useful tools for understanding and describing the teaching and learning of mathematics.

One of the basic concepts in mathematics is integers. The concept of integers is considered a very important prerequisite for mathematics. Therefore this concept is taught since elementary school, so that students can follow the next material (higher level) provided with a good understanding of the concept. The concept construction process in integers is very dependent on the meaning of the sign or symbol (Bofferding, 2014; Vlasis, 2008). This is also supported by several other studies regarding students' interpretations of symbols in mathematics which have received special attention. Among them are the meaning of symbols " $+$ " on negative integers, students can treat the negative sign of addition as a sign of subtraction (i.e., solve $-5+6$ as $6-5$) (Bofferding, 2010), or Vlassis (2008) which revealed three main meanings of symbols " $-$ " is unary which means the minus sign attached to the number, binary, the meaning of the minus sign according to the subtraction operation, and symmetrical or the opposite meaning of the minus sign indicates the multiplying operation by -1. Eichhorn (2018) discussed the meaning of the symbol "=" which has been interpreted by students as "do something" or search for the result of the left-hand side and write it after the "=" sign (i.e. $4+6=10$), whereas according to him it is broader than that, the symbol "=" has the meaning of equality (meaning of the relation). Students who understand the symbol "=" as an operation, when faced with a problem $7+8=\ldots+5$ would think 15 is the correct value to fill in the points as a result of $7+8$. Students who answered this did not interpret the sign "=" as a relation. Meanwhile, students who understand the meaning of the relation will think that on the left side there is the sum of the numbers 7 and 8 and on the right side there is already the number 5, so the blank space shall be filled in with 10. The signs discussed in this study involve negative signs and representations that are produced and used by students in solving problems.

Understanding the concept of mathematical material is very dependent on the representation and interpretation of signs (Mudaly, 2014). The material in integers contains signs and understanding, and the mastery of the concept depends on the interpretation of the signs involved (Bofferding, 2014). Correct interpretation will bring deep understanding to students. But in fact, students still have difficulty understanding signs in integers (Bishop, 2014b; Gallardo, 1995): for example, in negative or positive signs (Vlasis, 2008; Bofferding 2014), signs in number sequence relations (Schindler, 2017; Bishop et al. 2014ab), signs of operation (Eichhorn, 2018; Bofferding, 2010), as well as signs in representation (Widjaja et al. 2011; Bruno & Cabrera, 2005). Difficulty in understanding these signs has an impact on students' incorrect understanding of the concept of integers (Akhtar, 2018) and will certainly affect student performance in the problem solving process which results in errors (Khalid &Embong, 2019). These studies provide an overview of the relationship between a sign or symbol and its meaning construction.

The importance of the role of sign interpretation in achieving conceptual understanding has been shown through several studies on various student interpretations of signs in the integer concept. However, previous researches which focused on Peirce's triadic were still dominated by the topic of geometry (Daher, 2014; Alshwaikh, 2010; Palayukan, et al, 2020), besides functional graphs (Mudaly, 2014;) and pattern growth (Miller, 2015). Whereas in mathematics integers are topics that require in-depth attention, because understanding the concept is very dependent on the meaning of the signs involved in it (Bofferding, 2014; Vlasis, 2008). Therefore, it is necessary to make an effort to trace the process of students interpreting signs in order to obtain an overview of students' thinking processes and to be able to provide appropriate treatment for students in understanding integers. In addition, our study attempts to contribute to expanding semiotic research in various fields of mathematics through the chain of relationships between signs - integers - semiosis. To achieve this goal we used the Peirce semiotic lens known as triadic (Representamen, Object, and Interpretant) as a tool to reveal students 'thinking processes in mathematics, especially cognitive processes related to students' interpretation of signs in solving integer problems.Based on the previous explanation, this study aims to answer the problem: how can the semiosis connections occour in problem solving?

Method

Research Model

This research is a descriptive exploratory study with a qualitative approach. The descriptive exploratory method is used to describe, analyze and interpret the students' meaning of the signs. While the qualitative approach is ideal to use because it allows investigation on students' interpretation and cognitive construction to describe students' semiotic process (semiosis) in interpreting signs in the concept of integers.

Participants

To 32 students of class VII SMP Negeri (State Junior High School) in South Sulawesi, they were given problems and interviewed regarding solving integer problems. The results were grouped based on semiotic characteristics, and obtained5students who tended to use iconic and symbolic representamen. The first subject (S1) was selected from 2 subjects who were consistently solving problem using iconic representamen, and the second subject (S2) was selected from 3 subjects who were consistently using symbolic representamen.

Data Collection Tools

The instruments in this study were questions and interview sheets. The questions aim to provide opportunities for students to be able to solve problems in several ways or representations according to the knowledge they have. The results of the written work obtained from these questions were used as

1. Sort the following integers from smallest to largest and give a reason! $2, -3, 0, -9, 3, 8, -6$.
2. An amateur diver firstly practices diving at 2 meters depth below sea level. After getting used to it, then he descended again to the 5 meters depth below sea level. What is the difference of depth in those two conditions?
3. Rani walks 20 meters towards the west and then turn around to walk to her first place for 14 meters. Where and in what direction she is going compared to her first starting

Figure 2. Test instrument

a first step to see and investigate the subject's thinking process (As'ari et al. 2019).
The stages of data collection were carried out by giving 3 question items to 32 students, analyzing answers, determining subjects based on criteria, interviewing as a triangulation stage, analyzing the results of triangulation, and drawing conclusions.

Data Analysis

The stages of data analysis are (1) examining the test results and interview transcripts, (2) reducing the data, (3) categorizing/coding, (4) determining and describing the students' semiotic process, (5) drawing conclusions. The analysis process is carried out through an emphasis on the semiotic analysis of Peirce's theory which views that semiosis (the process of interpreting signs) is a relation between triadic/ trichotomy between signs or representamen (R) - object (O) - interpretant (I). Based on Peirce's semiosis view, the indicators for each triadic component to be used in the analysis in this study are as follows.

Table 1. Semiosis Components

Peirce's Semiotic Components	Definition	Indicators
Representamen	The physical existence of something that is used to represent something other than the sign (in other words the form of the sign)	• displays a concept (object) verbally / symbolically and visually • conveys the meaning of the thing that is represented • relates the properties of the sign relation in integers • gives meaning to the integer sign and its relations with other signs
Object	The concrete object or concept that the sign refers to	• the concept represented by the representamen • concepts that are thought out while producing representamen
Interpretant	Mental concept which is the meaning that arises from the comprehension/understanding /response/reaction /perception of the sign	• remembers the concept based on representamen which has been produced. • connects representamen seen with the concept (object / subject matter) accordingly • makes decisions in the form of meaning according to the understanding and perception of the existing signs (representamen)

Results

The results of interpretations made by the subjects towards the signs they encounter and they produce in the process of solving problems are grouped according to representamen, object, and interpretant based on Peirce's semiotic theory.

The subject of the iconic representamen

In finishing problem 1, Subject S1 used the help of a number line to find a sequence of numbers $2, -3, 0, -9, 3, 8, -6$. Through this number line representation, S1 had an understanding that the number on the left is smaller than the number on the right and can be sequenced based on the position of the $+$ numbers and the $-$ numbers on the number line. So he concluded that the more to the right position, the greater the number value and the more to the left position, the smaller the number value.

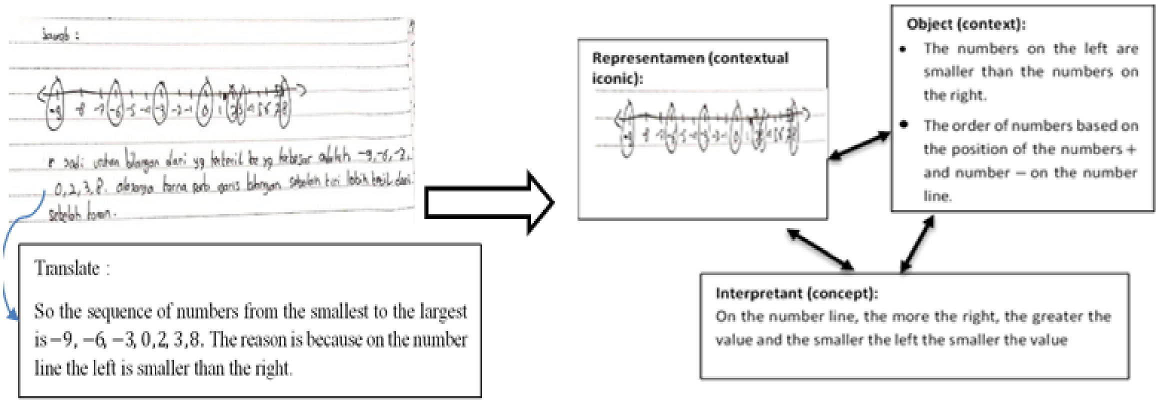

Figure 3. Analysis of semiosis in problem 1

For problem 2, through the number line S1 represented the below sea level position as negative and in the left side of zero, while the above sea level position as positive and is positioned on the right side. The starting point is located 2 meters under the sea which is represented by -2, and the second condition is 5 meters below sea level, represented by -5. Therefore the difference between the two diver positions is the number of integers that are in between -2 and -5, that is 3 m.

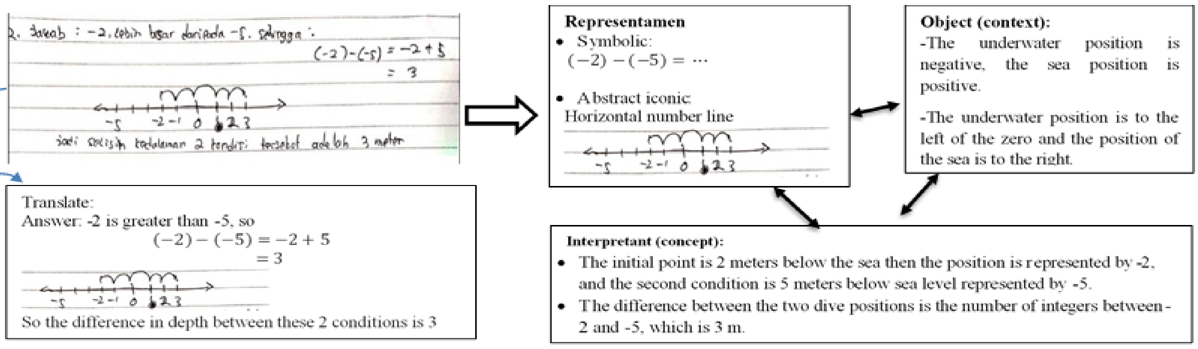

Figure 4. Analysis of semiosis in problem 2

On problem 3, according to S1 the west direction means to the left of the number zero and the reversed means to the right from zero on the number line. The original position is zero. On the number line, walk as far 20 meters to the west means negative value such as -20. Because it reverses direction, Rani's position will go backwards, which means going to the right as far as 14 meters that is 14 number. Therefore, the final position is at 6 meters on the left, which is in the west.

The following is an excerpt from the interview results regarding S1's meaning of signs (R: researcher

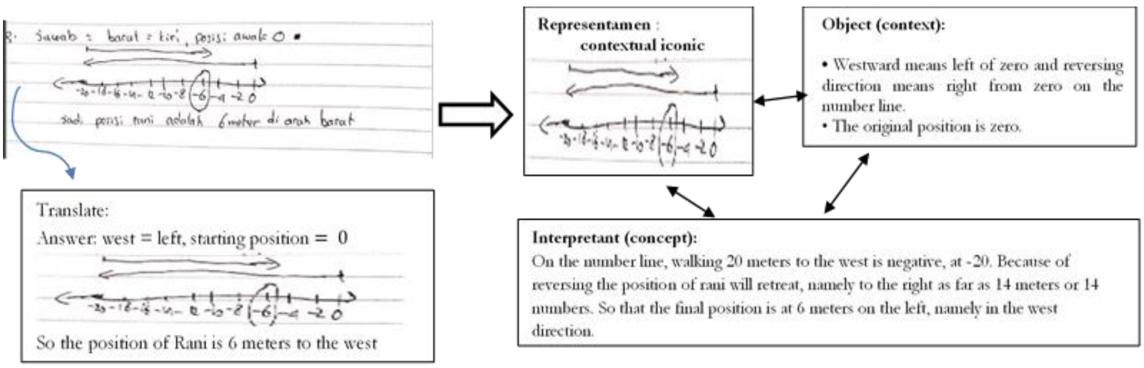

Figure 5. Analysis of semiosis in problem 3

and S1: subject 1).

R: Ok. Now try to look at each of the three questions given. What first steps do you take or perhaps you think about when working on the problem?
S1: When I read the problem, I immediately remember the number line that I had studied before, Ma'am.
R: Why a number line? Try to explain to me!
S1: It's this way, Ma'am: question number 1 told me sort out numbers, In order to do that, I use this number line Ma'am (pointing to the results of the work).
R: Oh I see. Then what do you mean by the number line (pointing to the number line that the student made on each question number)?
S1: Hmmm ... this one (number 2) I also use a number line similar to number 1 Ma'am. So that I can distinguish the above sea level (pointing to the right) and below sea level (pointing to the left). It's easier in my opinion Ma'am .... number 3 is the same Ma'am ... west is to the left (points to the west).
....
R: Then how to find the result?
S1: I'm actually confused Ma'am.. (hehehe).
R: Confused?? Mmmm... But yesterday you could answer, what did you think of?
S1: Yes Ma'am.. But this one is clear Ma'am (pointing to number line no. 1). Positive means the one to the right, the negative to the left. So I wrote down the number in the problem, then I answered the sequence. Ehmm I mean I adjusted this position Ma'am.
R: So this is the final answer?
S1: That way, it means that it matches Ma'am (pointing to the final answer), so this number line to the left is negative, so it is smaller than the right, Ma'am, which is positive. From what I remember, the numbers are getting larger from left to right Ma'am.
....

Subject S1 consistently used iconic representamen either the representation is in accordance with the context of the problem (contextual iconic) or abstract iconic representamen (out of context). After **observing** problem as the initial representamen, the first to come in mind of S1 was the visual model (object) about what is discussed in the problem, then the subject connected the number sequence, the position of the diver and the direction of each problem with a number line (interpretant), at this stage direct interpretant occurs. Then S1 **manifested** sign (which previously was an object in mind) to be the new sign (representamen) which fits the context of the problem. At this stage, S1 produced a sign in the form of a horizontal number line which we call contextual iconic in the number sequence case (Figure 3), as abstract iconic in the case of sea depth (Figure 4) and as contextual iconic in the case of direction (Figure 5). Furthermore, in order to obtain interpretant, S1 connected representamen with object accordingly. Based on the representamen made, S1 performed **confirmation** by connecting representamen (sign) " $+$ "with object "Right side" or "above the sea position" and representamen (sign) " $-$ "with object "Left side" or "below the sea position". After obtaining temporary interpretant using the previously made connection, S1 performed **reflection** by connecting all representamen that has been made in the beginning with object to get the final interpretant. Its purpose is to ensure the answer obtained.

The subject of the symbolicrepresentamen

In solving problem 1, to find a sequence of numbers $2, -3, 0, -9, 3, 8, -6$ Subject 2 used symbolic understanding. He thought the value of a number based on positive or negative signs and the number's magnitude. The larger the number with a positive sign the greater the value. The larger the number with a negative sign, the smaller the value.

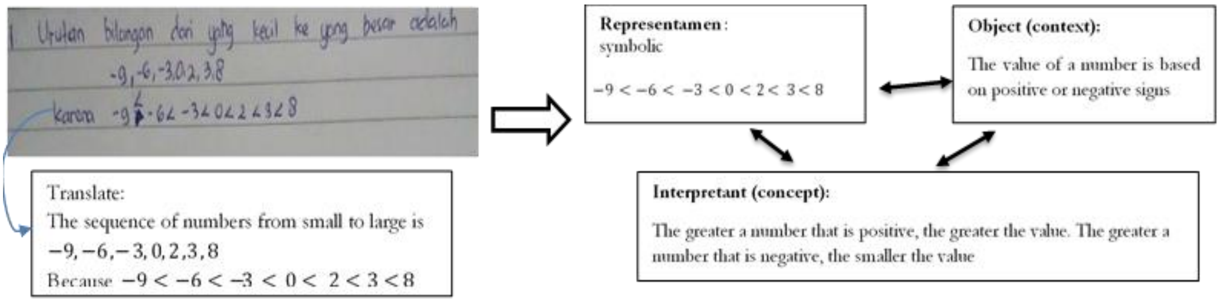

Figure 6. Analysis of semiosis in problem 1

In problem 2, through the symbols he produced, in order to be able to solve the problem, S2 used two interpretations. First, subject used the concept of distance from one point to another. According to the subject, because both conditions are below sea level, the difference requested is the same as the distance from 2 to 5 in conditions below sea level (Figure 7). Secondly, the subject used the concept of increment/ substraction operation. In this view S2 understood that the difference in

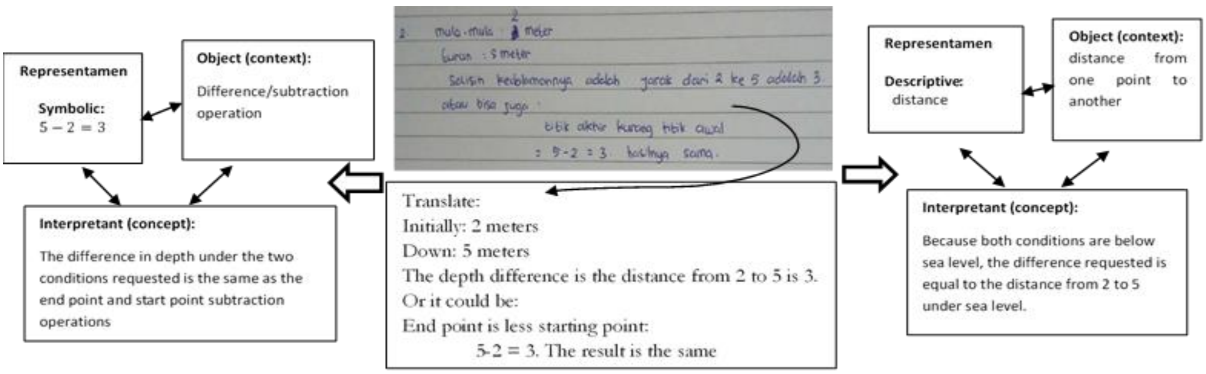

Figure 7. Analysis of semiosis in problem 2

depth under the two conditions is equal to the subtraction operation of the end point and the starting point.

For problem 3, reversing direction is tantamount to performing a subtraction operation. Rani's travel distance to the West is farther than the reversal distance. Reverse direction instructs to perform a subtraction operation because it requires moving backwards from the original position. Then the position is determined by completing $20 - 14 = 6$. Meanwhile, for the direction, it is determined by considering that the distance of Rani's journey to the west is farther than the distance to reverse

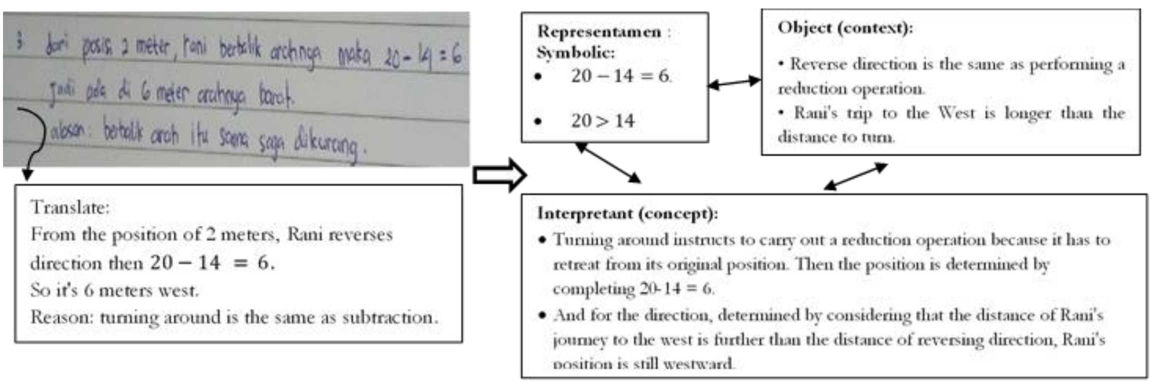

Figure 8. Analysis of semiosis in problem 3

direction, so Rani's position is still to the west.

The following is an excerpt from the interview results regarding S2's meaning of signs (R: researcher and S2: subject 1).

R: Ok. Now try to look at each of the three questions given. What first steps do you take or perhaps you think about when working on the problem?

S2: From the problem, what I noticed was that the negative sign and the number, Ma'am.

R: What do you see from this sign (pointing to a sign −)? Try to explain to me!

S2: Later it will be different when there is a negative sign or not. So this (number 1), if I want to sort it, I need to pay attention to the sign and the numbers, Ma'am. This is the largest (number 9) but it is negative so it is the smallest Ma'am. So the order is using this sign (pointing to a sign <) so it can be sorted.

....

R: Ok fine. Then for number 2, what does the distance and end point mean?

S2: Oww for that (number 2), it starts at 2, then drops by 5 so it's the same as the distance, Ma'am. 2 to 5 means 3.

R: What about this? (pointing to S2's second alternative answer)

S2: Oo, I remember there is one more way, Ma'am, so I wrote two answers. The difference can be found by subtracting, Ma'am.

R: What about number 3 then?

S2: For number 3, it asks for the position, Ma'am, and there's a word saying "reverse", so it had to be substracted ma'am.

R: Why should it be subtracted?

S2: This is 20 meters from it, uh, this one (referring to the answer that is written as 2) is not 2 Ma'am, it should be 20. Because reversing means moving backwards, so it's automatically subtracted. So the answer is 6 Ma'am.

R: So are you sure of your answer?

S2: Yes Ma'am, after re-checking, it was correct (pointing to all the answers) with what was asked in the questions.

....

Subject S2 consistently used symbolic representamen at each completion. By observing problem as initial representamen, the first thought is the number value and number operations (object). Furthermore S2 connected the number sequence of diver position and direction with the numeric value symbol (positive or negative) and operation. This is a process of acquiring interpretant directly. The next stage is S2 **manifested** sign (which previously was an object in mind) to be a new sign (representamen) according to the problem. In the case of number sequences and directions (Figures 6 and 8) subject used symbolic representamen, for problem 2 subject made symbolic and descriptive representamen (Figure 7). Based on the representamen that has been made, in order to obtain interpretant, S2 connected representamen with object accordingly. S2 conducted **confirmation** on problem 1 by connecting representamen (sign) "+" with object "larger" and the sign " − "with "smaller". In problem 2 the subject connected representamen (sign) "difference" with the object "distance" and "subtraction operation". In problem 3 the subject connected representamen (sign) "reverse direction" with object "subtraction operation". After obtaining temporary interpretant using the previously made connection, S1 performed **reflection** to ensure the answer obtainedby connecting all representamen that was created from the beginning with object to get the final interpretant.

Discussion and Conclusion

In the problem solving process, S1 and S2 interpreted the initial representation (initial R), or problems, with different meanings. S1 consistently used iconic representations (contextual or abstract), and S2 consistently used symbolic representations. When discovering new concepts, students will try to apply their previous knowledge in an effort to understand the new knowledge or information. Especially in the introduction of integers and their operations, students who are recently introduced to integers can interpret the signs they encounter in various ways (representations). Representations are used to explain, describe, and develop mathematical ideas and even to solve problems (Daryaee, 2018). Likewise, subject S1 and S2 used different representations when solving problems. S1 used representations in the form of images/ visuals (number lines), each of which is adjusted to the context in the problem. Such a representation according to Peirce's theory is an iconic representation, which is a sign that is similar to the object it represents. However, S1 used a representation that is not contextual in problem 2 by using a horizontal number line. Meanwhile, the context is the depth of the sea and it is possible to use a vertical number line. Therefore, we divide the category of iconic representations into contextual iconic if it fits the context of the problem and abstract iconic if it is out of context. In addition to that, there was a type of representation found in this study made by S2, which is the symbolic representation. In this case S2 used symbols in the process of finding mathematical truth (Kurniati et al. 2018. We see this as a symbolic representation, because it is presented without supporting images/ visuals.

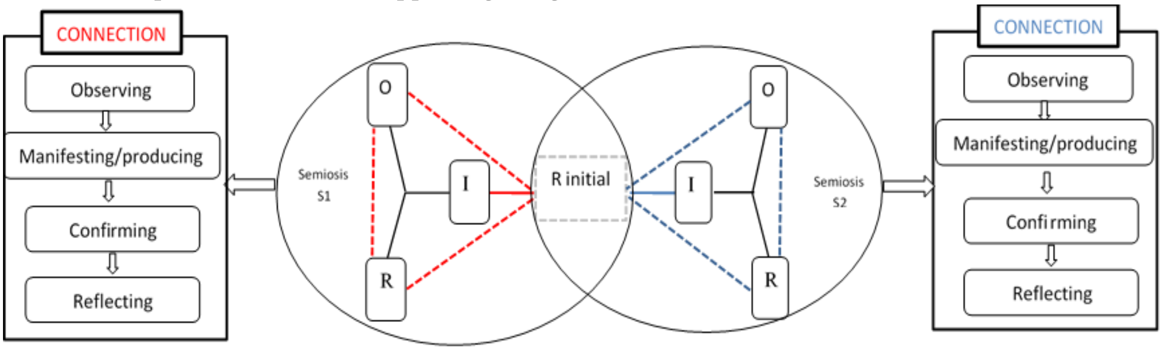

Figure 9. Semiosis connection

Furthermore, at each stage of semiotics, the triadic components (Object, Representamen, Interpretant) are connected to each other (see Figure 1) and occur infinitely (Eco, 1990). Each interpretant will become a new representative for the next process. We call that relationship as "connection". The intended connection is subjective. This means that it depends on the object in mind (Godino et al. 2011) and how each individual uses mathematical knowledge to lead them to a specific interpretation (Steinbring, 2006). Therefore the object becomes a determining factor and plays an important role in the occurrence of this "connection" process. The way to get a deep understanding of a concept in mind is through the ability to represent a concept in multiple representations (Ozel et al. 2008). The connection process in semiotics begins with **observation** on the initial representamen. According to Schreiber (2013) this observation process produces objects. Both subjects made observations of any information contained in the problem. The difference between S1 and S2 observations lies in the object in mind which is directly connected with representamen. S1 thought of the visual object seen in the answer by always using a number line, while S2 thought of the symbolic object that was visible when he tended to use symbols in solving. The immediate impact of the object on the subject's mind is a new sign **manifested** in the form of a representamen. This can be seen from the representamen produced by each subject. S1 produced estimates of the placement of numbers on the number line, attempts to show numbers by marking a circle (figure 3), with a jump of numbers (figure 4), with an arrow indicating position and direction (figure 5). Meanwhile, S2 produced symbols in the form of signs $<$ to give the signal "smaller value" (Figure 6), in the form of a sign $-$ to indicate "difference/ operation" (figure 7) or to indicate "reverse direction" (figure 8). This is supported by the opinion of Sáenz-Ludlow &Kadunz (2016) which stated that mathematical notation used in mathematical situations can affect the character of the sign within the translator/ interpreter's mind. This means that the notation they use is influenced by the type of sign they think of.

Confirmation stage appears when the subject again relates the representation to the corresponding object. Both S1 and S2 reconnected the representations they produced with the object they were thinking of in order to obtain the appropriate interpretant. S1 connected (see figures 3 and 4) representamen (sign) "+" with the object "Right side" or "position over the sea" and representamen (sign) "−" with the object "Left side" or "underwater position". Meanwhile S2 connected the representamen (sign) "+" with the object "larger" and a sign "−" With "smaller" (figure 6), then related the "difference" representamen (sign) to the object "distance" and "subtraction operation" (Figure 7). In problem 3 the “reverse” representation (sign) was connected with the “subtraction operation” object (Figure 8). In the end both subjects did **reflection** by connecting all the representamen that have been made from the beginning with the object to arrive at the final interpretant, and this was done as a step to ensure their final answers are related to the meaning process of signs in the concept of integers (revealed in the interview session of the two subjects).

Our results illustrate that mathematics and signs are involved in a relation, which is the meaning. The meaning relation in integers depends on signs, both signs that are observed, and signs produced by students. This knowledge is mediated between the context of the sign and reference/ object. The form of signs in this study becomes visible for example when students write, draw, read, and show.

Semiotic connection is a cognitive process of how students seek and connect with the meaning that they construct themselves to reach an understanding. There are four stages of a semiotic connection that involve triadic components (representamen, object, interpretant): observing (Representamen-Object-Interpretant connection), manifesting (Object-Representamen-Interpretant connection), confirming (Representamen-Object-Interpretant connection) and reflecting (Interpretant-Representamen-Object connection). The three components in semiotics are connected to each other in an infinite cycle. Overall strong connections between components will bring precise and deep understanding to students. Besides that, our study found that object is a determining component in carrying meaning and directing students to certain cognitive processes. On the other hand, this study also provides results about the components of semiotics which can be divided into several types, which are representamens: symbolic representamen, and iconic representamen; dynamic objects; and interpretants: direct, temporary, and final.

Recommendations

The results of this study provide an overview of the connection or semiosis relationship between components in the sign meaning process that can be used by other researchers as a basis for investigating students' ways of constructing meaning, especially signs in mathematics. In addition, we suggest that further research can reveal students' semiotic processes in other materials or concepts as well as in different levels of education (class) to obtain broader data.

Disclosure and Conflicts of Interest

This research is an original work and does not contain defamation or violation of law or violation of other people's rights or privacy. The authors declared that there is no potential conflict of interest regarding research, authorship, and/or publication of this article.

Acknowledgement

In this study, all authors have the same contribution and according to their field of expertise.

References

Akhtar, Z. (2018). Students' Misconceptions about Locating Integers and Decimals on Number Line. International Journal of Innovation in Teaching and Learning (IJITL), 4(1).

Alshwaikh, J. (2010). Geometrical diagrams as representation and communication: A functional analytic framework. Research in Mathematics Education, 12(1), 69-70.

As'ari, A.R., Kurniati, D., &Subanji. (2019). Teachers Expectation of Students' Thinking Processes in Written Works: A Survey of Teachers' Readiness in Making Thinking Visible. Journal on Mathematics Education, 10(3), 409-424. https://doi.org/10.22342/jme.10.3.7978.409-424.

Berger, M. (2006). Making mathematical meaning: from preconcepts to pseudoconcepts to concepts. Pythagoras, 63,14–21.

Bishop, J. P., Lamb, L. L., Philipp, R. A., Whitacre, I., &Schappelle, B. P. (2014a). Using order to reason about negative numbers: the case of Violet. Educational Studies in Mathematics, 86(1), 39-59.

Bishop, J. P., Lamb, L. L., Philipp, R. A., Whitacre, I., Schappelle, B. P., & Lewis, M. L. (2014b). Obstacles and affordances for integer reasoning: An analysis of children's thinking and the history of mathematics. Journal for Research in Mathematics Education, 45(1), 19-61.

Bofferding, L. (2010, October). Addition and subtraction with negatives: Acknowledging the multiple meanings of the minus sign. In Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (Vol. 6, pp. 703-710).

Bofferding, L. (2014). Negative integer understanding: Characterizing first graders' mental models. Journal for Research in Mathematics Education, 45(2), 194-245.

Bruno, A., & Cabrera, N. (2005). Una experienciasobre la representaciónen la recta de númerosnegativos. Quadrante: Revistateórica e de investigação, 14 (2), 25-41.

Daryaee, A., Shahvarani, A., Tehranian, A., Lotfi, F. H., &Rostamy-Malkhalifeh, M. (2018). Executable Functions of the Representations in Learning the Algebraic Concepts. PNA 13(1), 118.

Daher, W. M. (2014). Manipulatives and problem situations as escalators for students' geometric understanding: a semiotic analysis. International Journal of Mathematical Education in Science and Technology, 45(3), 417-427.

Dimmel, Justin K. & Herbst, Patricio G. The Semiotic Structure of Geometry Diagrams: How Textbook Diagrams Convey Meaning.Journal for Research in Mathematics Education, Vol. 46, No. 2 (March 2015), pp. 147-195.

Eco, U. (1990). Drift and unlimited semiosis.

Eichhorn, M.S., Perry, L.E., &Brombacher, A. (2018). Students' early grade understanding of the equal sign and non-standard equations in Jordan and India. International Journal of Research in Education and Science (IJRES), 4(2), 655-669. DOI:10.21890/ijres.432520.

Gallardo, A. (1995). Negative numbers in the teaching of arithmetic: Repercussions in elementary algebra. In D. Owens, M. Reeds, & G. Millsaps (Eds.), Proceedings of the Seventeenth

Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (pp. 158–163). Columbus.

Godino, J. D., Font, V., Wilhelmi, M. R., &Lurduy, O. (2011). Why is the learning of elementary arithmetic concepts difficult? Semiotic tools for understanding the nature of mathematical objects. Educational Studies in Mathematics, 77(2-3), 247-265.

Hoed, B. H. (2002). Strukturalisme, Pragmatik dan Semiotikdalam Kajian Budaya. dalam Indonesia: Tanda yang Retak.

Hoed, B. H. (2014). Semiotik&DinamikaSosialBudaya.

Khalid, M., &Embong, Z. (2019). Sources and Possible Causes of Errors and Misconceptions in Operations of Integers. International Electronic Journal of Mathematics Education, 15(2), em0568.

Kriyantono, Rachmat. (2007). Teknik PraktisRisetKomunikasi. Jakarta: Kencana

Kurniati, D., Purwanto, As'ari, A.R., &Dwiyana. (2018). Exploring the Mental Structure and Mechanism: How the Style of Truth-Seekers in Mathematical Problem Solving?.Journal on Mathematics Education, 9(2), 311-326.

Littlejohn, S. W., & Foss, K. A. (2009). Encyclopedia of communication theory (Vol. 1). Sage.

Miller, J. (2015). Young Indigenous Students' Engagement with Growing Pattern Tasks: A Semiotic Perspective. Mathematics Education Research Group of Australasia.

Morgan C. (2014) Mathematical Language. In: Lerman S. (eds) Encyclopedia of Mathematics Education. Springer, Dordrecht.

Mudaly, V. (2014). A Visualisation-based Semiotic Analysis of Learners' Conceptual Understanding of Graphical Functional Relationships. African Journal of Research in Mathematics, Science and Technology Education, Vol.18, No.1, 3-13,doi.org/10.1080/10288457.2014.889789.

Ozel, S., Yetkiner, Z. E., &Capraro, R. M. (2008). Technology in K-12 mathematics classrooms. School Science and Mathematics, 108(2), 80-85.

Palayukan, H., Purwanto, Subanji, &Sisworo. (2020). Student's semiotics in solving problems geometric diagram viewed from peirce perspective. In AIP Conference Proceedings (Vol. 2215, No. 1, p. 060020). AIP Publishing LLC.

Peirce, C. S. (1991). Peirce on signs: Writings on semiotic. UNC Press Books.

Pino-Fan, L. R., Guzmán, I., Font, V., & Duval, R. (2017). Analysis of the underlying cognitive activity in the resolution of a task on derivability of the absolute-value function: Two theoretical perspectives. PNA. Revista de InvestigaciónenDidáctica de la Matemática, 11(2), 97-124.

Presmeg, N., Radford, L., Roth, W. M., &Kadunz, G. (2016). Semiotics in mathematics education (p. 40). Springer Nature.

Quinnell, L., & Carter, M. G. (2012). Greek or not: The use of symbols and abbreviations in mathematics. Australian Mathematics Teacher, 68(2), 34-40.

Radford, L. (2003). Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generalization. Mathematical thinking and learning, 5(1), 37-70.

Sabena, C., Radford, L., &Bardini, C. (2005). Synchronizing Gestures, Words and Actions in Pattern Generalizations. International Group for the Psychology of Mathematics Education, 4, 129-136.

Santi, G. (2011). Objectification and semiotic function. Educational Studies in Mathematics, 77(2-3), 285-311.

Sáenz-Ludlow, A., &Kadunz, G. (2016). Constructing knowledge seen as a semiotic activity. In Semiotics as a tool for learning mathematics (pp. 1-21). Brill Sense.

Schindler, M., Hußmann, S., Nilsson, P., & Bakker, A. (2017). Sixth-grade students' reasoning on the order relation of integers as influenced by prior experience: an inferentialist analysis. Mathematics Education Research Journal, 29(4), 471-492.

Schreiber, C. (2013). Semiotic processes in chat-based problem-solving situations. Educational Studies in Mathematics, 82(1), 51-73.

Steinbring, H. (2006). What makes a sign a mathematical sign?–An epistemological perspective on mathematical interaction. Educational studies in mathematics, 61(1-2), 133-162.

Torigoe, E. T., & Gladding, G. E. (2011). Connecting symbolic difficulties with failure in physics. American Journal of Physics, 79(1), 133-140.

Vile, A. (1993). Is this a sign of the times? A semiotic approach to meaning-making in mathematics education. Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Vlassis, J. (2008). Peran simbolmatematikadalampengembanganangkakonseptualisasi: Kasustanda minus. PsikologiFilsafat , 21 (4), 555-570.

Widjaja, W., Stacey, K., &Steinle, V. (2011). Locating negative decimals on the number line: Insights into the thinking of pre-service primary teachers. The Journal of Mathematical Behavior, 30(1), 80-91.

Zayyadi, M., Nusantara, T., Hidayanto, E. Sulendra, I.M., &Sa'dijah, C. (2020). Content and Pedagogical Knowledge of Prospective Teachers in Mathematics Learning: Commognitive Framework. Journal for the Education of Gifted Young Scientists, 8(1), 515-532. DOI: http://dx.doi.org/10.17478/jegys.642131.