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Dynamics of Classroom Misconception: An Analysis Using Mathematical Modeling and Simulation

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Abstract

Mathematical modeling technique was used to simulate classroom misconception dynamics in a binary perception system with the effects of student interaction, in two different conditions: with learned student influence and without. A Compartmental model (SIR of Epidemiology) was used as a baseline theory in generating the Misconception Dynamics Model (MDM) for the simulation and analysis. The presence of influential learned students in a classroom interaction greatly impacts the misconception dynamics of a classroom system. The inclusion of misconception correction by fellow students in an open class interaction environment drastically affects the time evolution of the spread of misconception by slowing its spread and by direct conversion of misinformed (M) and unlearned (U) status to learned (L).

Keywords

Mathematical modelling; Misconception dynamics model; Simulation; SIR model

Introduction

Learning correctly the different concepts presented in the class is an essential thing to be ascertained. However, doing so is not something that could easily be realized as there are some concerns and issues that serve as barriers for the correct understanding of concepts. One is comprehension of the learners. Students' ability to understand is one of the many essential factors that account for the complete, substantial and rightful understanding of the different ideas, concepts and topics presented inside a class. Another is the misconception of the learners. It could not be doubted that misconception happens when students fail to correctly grasp the understanding of concepts.

Along this line, it is noted that misconception could be spread, because students in possession of a misconceived idea interact with other students. Interaction happens between and among students themselves, and the said interaction is facilitated sometimes by the conduct of classroom activities such as, but not limited to, grouping activities, collaboration works, pair tasks and many others. It could then be said that the spread of misconception inside a class is a legitimate concern needed to be addressed by educators. It is remarked along this line that it is an essential research activity to determine the spread of misconception. As from such a study, pedagogical implications could be drawn and empirical-based understanding could be used to mitigate the spread of misconception or put a halt to it.

Against this background, this study employed a mathematical modeling approach to develop models for the different simulation schemes to characterize and describe the dynamics of the spread of misconception inside a classroom. Misconception and the factors that affect its dynamics are one of the open topics in educational research. There are several theories in educational psychology that tries to explain the effect of classroom student interaction on conception of ideas of students in a classroom dynamic. In one social influence modeling conducted, it was observed that when an individual interacts with another, there is generally an increase in similarity between their opinions. This particular dynamic can also be tested for misconception spread within a classroom interaction. Although there are results that back assimilation as process in opinion formation, i.e., an individual conforms to the opinion of the majority, debate still exist because there are conditions that causes opinion differentiation, i.e., an individual tends to adopt an opinion different from the majority [1].

Previous studies on classroom misconceptions are mostly on identifying specific topics and subjects where students have a difficult time comprehending and then diagnosing and addressing these concerns through pedagogical methods [2] [3]. Some studies however try to navigate through the origin and development of misconception, like how students being active learners are influenced with what they do to the new information they are presented with as they construct their own knowledge [4]. One notable research done on this topic investigated the correlation of student interaction and increase of misconception. The results of this study suggest that the students who participated in collaborative activities in the traditional classroom had fewer science misconceptions than students who participated in collaborative activities in the online environment. Moreover, from pretest to posttest, the students in the experimental group increased in their science misconceptions [5].

Despite numerous studies done on misconceptions, the dynamics of it as a result of social interaction has not been widely studied by social scientists, educators, mathematicians and even computational scientists. However, there the has been a recent movement in physics to study social science, education and economics as a complex system. One emerging field trying to accomplish this is Sociophysics. This interdisciplinary study tries to explain social phenomena in the context of statistical physics and mathematical modelling. In a paper published about Models on Social Influence the researcher proposed that there is an urgent need for more theoretical work comparing, relating and integrating alternative models [6]. One of the tools used by physicists is Agent-Based Modeling, which is a computational model used to simulate interactions among independent agents with global rules on how they should interact. ABM is widely used in studying biological and social systems [7]. In one study conducted using ABM to investigate the interaction of students in a cooperative group with varied personality traits, the results suggested that a particular personality has a more favorable presence for a group to be successfully cooperating [8].

This study uses a mathematical modeling technique to analyze the dynamics of misconception in a classroom system with the presence of student interactions that can influence the propagation of fallacy in a fully connected network.

Modeling Misconception dynamics with student interactions

In Sociophysics, binary systems are the most frequently preferred due to their simplistic nature and thus are well-studied. Binary systems often eliminate the complexities brought about by multiple agents, parameters and decisions. There are several attempts to draw analogs of physical system and use them as a tool in modeling decision dynamics.

A classroom interaction can be deemed as a complex system due to the variety of possible decisions a single agent (student or a teacher) can prefer. Having these agents interact with one another to come up with a common outcome makes it challenging and interesting at the same time. The presence of modeling techniques to explore such complexities and simulate possible results of agent interaction or system behavior makes tasks that would have been impossible using empirical methods feasible.

In the context of studying and analyzing misconception dynamics in a classroom, numerous existing simulation models can be applied to imitate student interactions and the creation of prevalent system behavior or decision outcome. One of the most ubiquitous models that can be incorporated though used in the field of epidemiology is the SIR Model which is an example of a compartmental modeling technique.

1. The SIR Model

In 1989, Hethcote [9] introduced three basic models in epidemiology. One of them is SIR model. In the model, Hethcote [9] characterized the population (N) into three compartments: susceptible (S), infected (I), and recovered (R) or SIR. The number of individuals S, I, and R at time t is $S(t)$, $I(t)$ and $R(t)$ so the population at the time t is given by the equation:

$$N(t)=S(t)+I(t)+R(t) \tag{1}$$

In the model, the population is assumed to be constant so that the birth rate and mortality rate are equal to μ. Thus, the number of births is μN. If it is assumed that the born individual is a healthy but susceptible to disease, then the number of S increases by μN. Each compartment of S, I, and R has death so that the number of individuals in each successive compartment is reduced by μS, μI, and μR. The transmission disease can only occur through a direct contact between susceptible and infected individual. If β is the contact rate, then the number of S decreases by $\beta SI/N$ and the number of I increases by $\beta SI/N$. The compartment of I is possible to be cured and has permanent immunity. If γ is the recovery rate, then the number of I is reduced by γI and the number of R increases by a value of γI. Thus, SIR model by Hethcote [9] is:

$$\frac{dS}{dt} = \mu N - \beta \frac{SI}{N} - \mu S$$

$$\frac{dI}{dt} = \beta \frac{SI}{N} - \mu I - \gamma I \tag{2}$$

$$\frac{dR}{dt} = \gamma I - \mu R$$

With MS(0)≥0, I(0)>0I, R(0)≥0, and β, γ, μ>0. Model (2) is a first-order nonlinear differential equation system.

2. Analyzing SIR-Model

We will start with a mathematical *SIR-model* that will serve us as a benchmark for our *Misconception Dynamics Model (MDM)*. In the basic SIR-model, the flow between the three groups is: *S -> I -> R*. It is a one-way street where in the beginning most individuals are in the *S* group, eventually cascading via the *I* group into the *R* group. At each time step *'t'* a certain number of individuals are traversing from *S to I* and from *I* to *R*, while the total number of individuals *N* = *S*+*I*+*R* stays constant.

Originally according to Hethcote [9], the dynamics are governed by three variables *μ, β* and *γ*. However, for this particular analysis we will only include *β* which is the rate with which infectious individuals infect others and *γ* being the rate at which infectious individuals recover. We decided to ignore death and birth rates, thus eliminating *μ*. These dynamics are visualized below for a fixed *β* and *γ*:

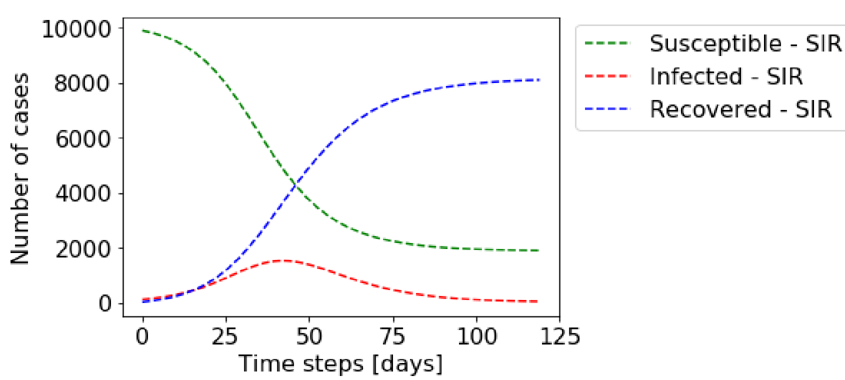

Figure 1. Time evolution of the number of cases using SIR Model

It can be observed that the number of infected individuals increases and peaks all the while the number of susceptible populations decreases significantly. After some time, the rate of infection slows. This is simply because by then a significant number of individuals already had the disease and have acquired immunity. Towards the end, the number of infected individual halts, eradicating the disease. With this simple SIR-model we can already observe some basic dynamics in the spread of infection or disease. This particular observation is what keeps our interest in the SIR Model.

The misconception dynamics within a class system can be compared to an epidemic dynamic in a population. This is because student interactions during class discussions or grouped activities can be possible opportunities for misconceptions to be transferred between misinformed and unlearned students. This paper explores the application of SIR Model to misconception spread in a class and analyzes the dynamics of information transfer between different types of students. To fit the existing model to our problem, certain parameters and variables would have to be altered with some assumptions.

3. The Misconception Dynamics Model

The Misconception Dynamics Model (MDM) is patterned from the SIR Model of Hethcote [9]. To study the misconception propagation within a class of population (N) we will define the students into three categories: unlearned (U), misinformed (M), and learned (L).

- ***Unlearned (U)*** are the students who are susceptible to misconceptions due to the lack of knowledge on the topic and openness to new information.
- ***Misinformed (M)*** are the students with misconceptions on the particular topic or subject
- ***Learned (L)*** are the students who are well-informed and are already knowledgeable or have acquired the right learning on the topic.

The number of students U, M, and L at time t is $U(t)$, $M(t)$ and $L(t)$ so the class population at the time t is given by the equation:

$$N(t)=U(t)+M(t)+L(t) \quad (3)$$

In this particular model, the class population is assumed to be constant. The transmission of information can be passed on in two ways: through a direct interaction between unlearned and misinformed students and between learned to unlearned and misinformed students. If β is the misconception rate between *U* and *M,* then the number of *U* decreases by $\beta UM/N$ and the number of *M* increases by $\beta UM/N$.

The population of *M* is possible to be corrected and would have a permanent learning thus eliminating the risk of further misconception on the topic. If α is the learning rate between *L* and *U* and *M*, then the number of L increases by $\alpha LU/N + \alpha LM/N$ and the number of *U* and *M* decreases by $\alpha LU/N$ and $\alpha LM/N$ respectively.

Thus, the *Misconception Dynamics Model* is:

$$\frac{dU}{dt} = -\beta\frac{UM}{N} - \alpha\frac{UL}{N}$$

$$\frac{dM}{dt} = \beta\frac{UM}{N} - \alpha\frac{LM}{N} \qquad (4)$$

$$\frac{dL}{dt} = \alpha\frac{UL}{N} + \alpha\frac{ML}{N}$$

with $U(0)\geq 0$, $M(0)>0$, $L(0)\geq 0$, and β, $\alpha>0$. Model (4) is a first-order nonlinear differential equation system.

4. MDM without Learned Student Influence: Case 1

This model will assume a classroom system randomly composed of N students on a fully connected network. There are three types of students in the system, each having a varied misconception status: learned, misinformed and unlearned. Although the location of a student in the classroom is random, the distribution of status can be set initially, i.e., 20% learned, 20% misinformed and 60% unlearned. Misinformed students may interact with uninformed ones and change their status depending on the 'β' value, while learned ones remain unaffected by the interaction. However, for this case, any interaction by learned with the unlearned will not influence the latter's status. Since the system is in a fully connected network, all agents are free to interact with one another until a popular status has been reached.

5. MDM with Learned Student Influence: Case 2

For this case we introduced the learned student influence on the other two type of agents. The model, for this case, allows a learned individual through the interaction to correct the misconception status of a misinformed student and to positively inform an unlearned one. The rules for influencing the status of the agents are similar to the previous rules, however the change in status as influenced by the learned agent is dependent on the ‘α’ value.

Implementation

The simulation was implemented in Python using Spyder IDE. The Python code integrated the equations in MDM (4) for student interactions by parameters ‘α’ = 0.2 and the ‘β’ = 0.2 values in a class size of N = 60 (a typical classroom population).

Although this part of the simulation will not be part of the model dynamics for misconception spread in a classroom system, the researcher decided to include a Case Zero where no misinformed students are present and no learned students have influence on unlearned ones to illustrate that the simulation works as governed by the model.

To simulate Case 1, the model started with a single misinformed student and the rest of the population being unlearned. Then we conducted a second simulation with 10 of the students being learned and the rest unlearned. The addition of learned agents for this case was deemed irrelevant since they will not be allowed to have any influence on the status change of the two other types of agents. Any number of ‘L’ students included in this simulation would be expected to remain constant. Phyton then runs the simulation and allows the interaction to progress until a popular status emerges, being misinformed or learned. As presumed, ‘unlearned’ would not be a possible popular status in the end of the simulation since all the unlearned agents would either be converted to learned or misinformed as dictated by the MDM.

To simulate Case 2, the model started with a one ‘M’ and ‘L’ student each and the rest of the population being ‘U’ agents. For this simulation, Learned agents are allowed to influence the change in the status of Unlearned and Misinformed agents. Then again, Phyton then runs the simulation and allows the interaction to progress until a popular status emerges as governed by the Misconception dynamics model.

Time evolution graphs of the different cases with parameter variations were generated for interpretation and analysis.

Results and Discussions

The simulations made for the different cases were all governed by the relationships as stated in Misconception Dynamics Model (MDM) in equation (4). Simulation results and the related discussion will be presented here.

1. Case 0: No Misinformed Students, No Learned Student Influence

Even though the result is already known, the case should be tested to assure that the simulation works. The following parameters are used for this case:

$$N = 60, U = 40, L = 20, M = 0, \alpha = 0 \text{ and } \beta = 0.1$$

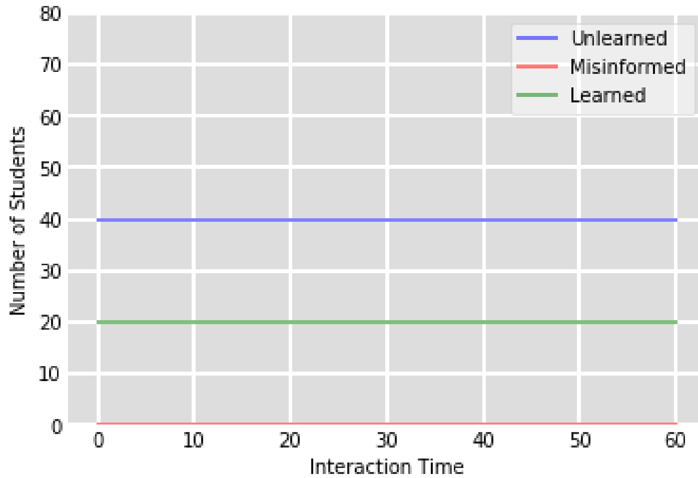

Figure 2. Time evolution of the misconception spread

As observed in Figure 2, the straight horizontal line for the graphs of unlearned and learned students, indicated a constant value over time. This depicts the absence of spread in misconception in the class. For the purpose of parameter discussion, we have arbitrarily assigned 0.1 as the value for the beta coefficient and zero for the alpha. The zero alpha value would ensure that learned (L) students though present in the population will not have an influence on the unlearned ones. To visualize the two graphs for 'U' and 'L' separately, we purposely chose U = 40 and L = 20.

2. Case 1: One Misinformed Student, No Learned Student Influence

Now we will observe how the system evolves if M = 1, with two sub-cases. (a) The first is set without any learned student, and (b) the second with L = 10. The rest have the following parameters for this case: N = 60, U = 59 and 49, $\alpha = 0$ and $\beta = 0.1$

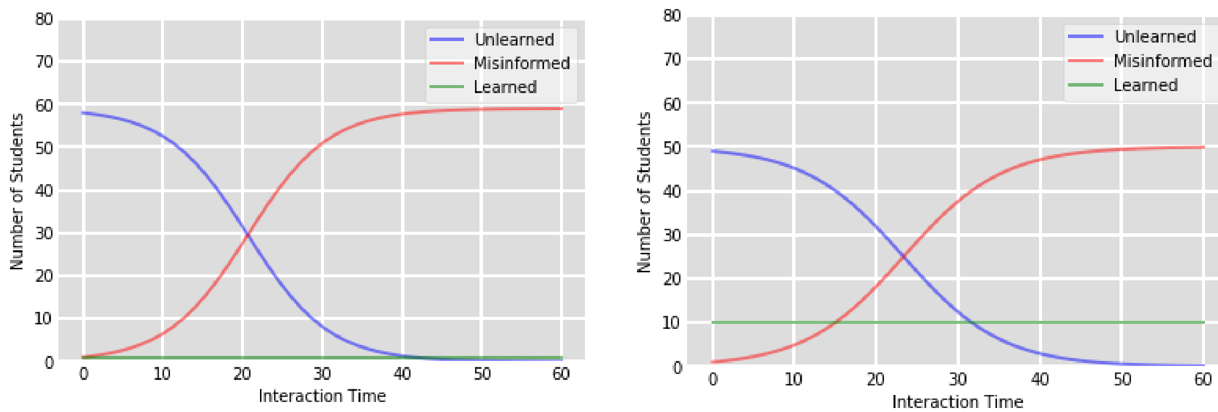

Figure 3. Time evolution of the misconception spread with misinformed (M) student and (left image) L = 0 and (right image) L = 10.

As depicted in Figure 3, the two images (left) and (right) show a proportional change in the number of unlearned population from misinformed students over time. It can be seen that the decrease in U is consequently due to the complete conversion of it to M. It should be noted that this type of behavior would only be true for completely susceptible unlearned students which this simulation presumed. In reality, students though uninformed of the topic or subject may choose to not be influenced by their classmates' misconceptions and would either maintain a status quo or be learned on their own. However, for the purpose of illustrating the dynamics of status change, the model stands.

In examining the effects of non-influential learned members of the class on status conversion of the other classmates, we can observe that the graph on the right image behaves exactly the same as the one on the left. We can deduce that even with the presence of Learned individuals, complete conversion of U to M still occurred. This was mainly due to the fact that we have assigned the model an alpha (α) value of zero, making the Learned individuals' interaction insignificant.

It is noteworthy however to discuss that though the misconception still spread to the entire class, but the presence of learned students within the population, slowed the progression significantly as we increase the population of L. This can be seen by the stretching of the saturation period in the image below, Figure 4.

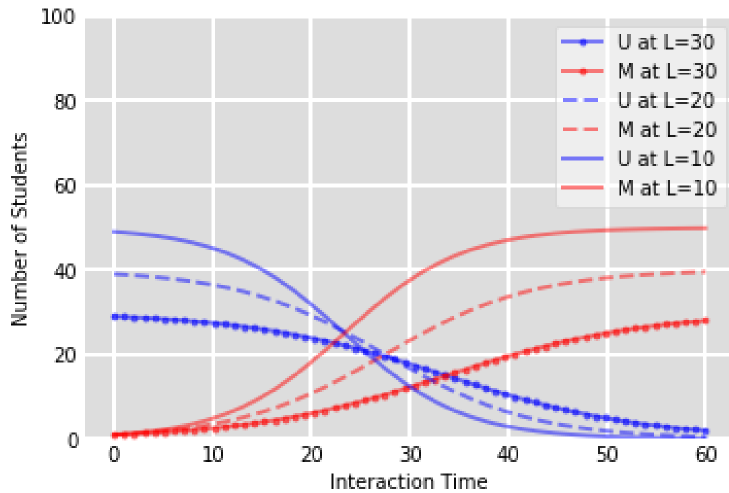

Figure 4. Time evolution of Misconception Propagation
with Increasing Number of Non-influential "L".

The observation above maybe an indirect impact of the non-influential L on the misconception progression by slowing the transfer of wrong information to the susceptible U population and by decreasing the chances of increasing M by conversion of U students. This phenomenon would have a very important implication in the analysis of the dynamics in misconception propagation in a classroom.

The mere presence, though without influence, of well informed and knowledgeable students in the classroom population has a negative impact on the spread of misinformation. This means that having a majority of the students gain the right understanding on the topic, eliminates the risk of misconception gaining the popular status. As we increase the number of Learned students through proper instruction, excellent pedagogical practices and better problem mitigation, we also slow down the possible proliferation of misconception in a class. Learned students in a class are like immune individuals in an epidemic. It would also be interesting to simulate the model with teacher involvement in the interaction to make the situation more realistic and wholistic.

3. Case 2: One Misinformed Student with Learned Student Influence

In a realistic classroom set-up, ignoring the impact of well-informed students on the propagation or elimination of misconception would be unreasonable and unwise. Now we will observe how the system evolves if M = 1, L = 1 and alpha (α) = 0.2. The rest have the following parameters for this case: N = 60, U = 58 and β = 0.2

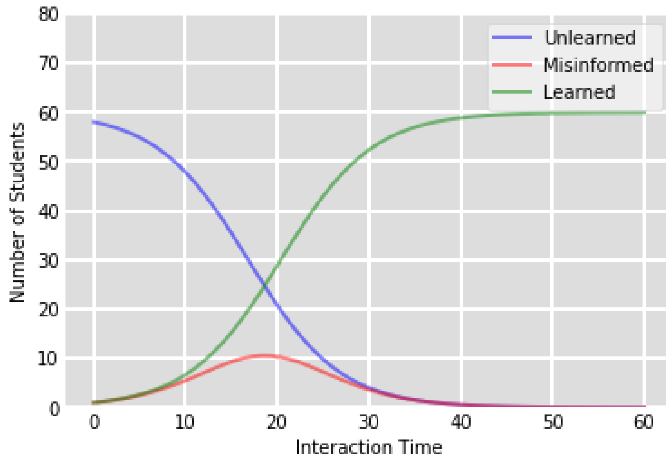

Figure 5. Time evolution of Misconception Dynamics with Influential Learned Population

The presence of influential learned students L as observed in Figure 5, drastically changes the time evolution of the propagation of misconception in a class. For case 1, the interaction of students would only produce a conversion of status between unlearned and misinformed members of the class. It can be seen that as one opinion increases the other decreases until a consensus is reached. However, in case 2 the integration of influential L individuals changes the dynamics of the system as interactions progress. This difference is due to the fact that the interactions between M and U agents with the L influential agents make it possible for a new type of conversion governed by the alpha (α) value or correction coefficient.

During these interactions students can now be converted to Learned ones as they interact with one another. As depicted in the graph, this will lead to a considerable decrease of unlearned individuals (blue graph) due to two possible conversions of population instead of one. This new type of conversion can also occur on the misinformed individuals, and thus stops the proliferation of M. It can be seen in the peaking of the red graph until it starts declining in number. Eventually, the simulation reaches a consensus where all the members of the population will be converted to Learned individuals (green graph).

The observations made above have profound implications to the dynamics of real classroom misconception dynamics and management. These tell us that the presence of educated and well-informed students would greatly affect the propagation of misconception in the class in two ways: first by correction of misconception on the misinformed population and second by direct instruction or information on the unlearned members of the class. This particular idea shows support on the study made by Wendt and Rockinson-Szapkiw on the effect of online student collaboration on misconceptions of students [5]. Their results suggested that the students who participated in collaborative activities in the traditional classroom had fewer science misconceptions than students who participated in collaborative activities in the online environment. It was possible that by student interactions the learned individuals had a positive effect on the spread of misconception.

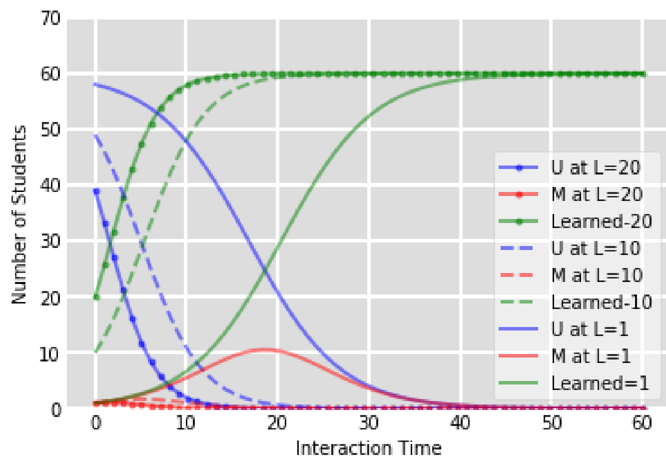

Figure 6: Time evolution of Misconception Dynamics with Increasing Number of learned students L

Time evolution depicted on Figure 6 shows the effects of increasing learned students on the misconception propagation in a class with one misinformed agent and a beta (β) value of o.2. As can be observed that the rise in the number of L at the beginning of simulation had a different effect as compared to Figure 4. Instead of slowing down the saturation period, the increase in the beginning L value sped up the consensus time, with the popular status being "Learned" at the end. This is probably due to the fact that in this simulation, the L individuals can now have a conversion effect on U and M, thus making the saturation quicker. Another notable effect is the lowering of the misconception peak. It can be observed that as we increase the starting L number, the maximum number of individuals that can become misinformed drastically changes.

Conclusions

Mathematical modeling technique was used to simulate classroom misconception dynamics in a binary perception system with the effects of student interaction, in two different conditions: with learned student (L) influence and without. A Compartmental model (SIR of Epidemiology) was used as a baseline theory in generating the Misconception Dynamics Model (MDM) for the simulation and analysis. The generated model has the form:

$$\frac{dU}{dt} = -\beta\frac{UM}{N} - \alpha\frac{UL}{N}$$

$$\frac{dM}{dt} = \beta\frac{UM}{N} - \alpha\frac{LM}{N} \qquad (4)$$

$$\frac{dL}{dt} = \alpha\frac{UL}{N} + \alpha\frac{ML}{N}$$

with $U(0)\geq 0$, $M(0)>0$, $L(0)\geq 0$, and $\beta, \alpha>0$. Model (4) is a first-order nonlinear differential equation system. From the formulated MDM, we can see that the spread of the misconception as indicated by an increase in M is proportional to the beta value. It is expected that a greater β and a high M would facilitate a higher and faster misconception spread. The presence of influential learned students in a classroom interaction, however, greatly impacts the misconception dynamics of a classroom system. The inclusion of misconception correction (α) by fellow learned students in an open class interaction environment drastically affects the time evolution of the spread of misconception by slowing its spread and by direct conversion of misinformed (M) and unlearned (U) status to learned (L). As expected, a greater α and a high L would result to lower and slower misconception spread until its eradication.

The results of the simulation and the formulation of a MDM in this paper, shows promise in studying, analyzing and possibly comparing misconception spread and elimination in a theoretical classroom where interaction of students can be controlled and set with empirical studies on a real classroom setting where student interactions together with other agents of learning like teachers and parents are more complex. Further investigation then would be a great addition to this information.

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