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Aws-R_k Spaces Where K ∈ {0, 1}

R. S. Suriya[1]* and T. Shyla Isac Mary[2]

[1]Research Scholar, (Reg. No – 18113112092021) Department of Mathematics, Nesamony Memorial Christian College, Marthandam, Tamil Nadu, India.

[2]Assistant professor, Department of Mathematics, Nesamony Memorial Christian College, Marthandam, Tamil Nadu, India.

Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627 012, Tamil Nadu, India.

Abstract – Aim of this paper is to introduce a new type of αws-R_k spaces.

Key words: αws- R_k spaces

INTRODUCTION

we introduce some new type of αws-spaces that is αws-R_0 spaces, αws-R_1 spaces by utilizing αws-closed and αws-open sets.

Definition 1.1: A space X is an αws-R_0 space if for every αws-open set contains the αws-closure of each of its points.

Theorem 1.2: A space X is an αws-R_0 space iff for every αws-closed set H, αws-cl ({x}) ∩ H = ϕ for all x ∈ X − H.

Proof:

Suppose X is an αws-R_0 space. Let H be αws-closed set in X. Let x ∈ X − H. Therefore X − H is αws-open. Since X is an αws-R_0 space, by using Definition 1.1, αws-cl ({x}) ⊆ X − H which implies that αws-cl ({x}) ∩ H = ϕ. Conversely, suppose for every αws-closed set H, αws-cl ({x}) ∩ H = ϕ for every x ∈ X − H. To prove X is an αws-R_0 space. Let V be an αws-open set such that x ∈ V. Therefore X − V is αws-closed. By our assumption, αws-cl({x}) ∩ (X − V) = ϕ ⟹ αws-cl({x}) ⊆ V. Then by using Definition 1.1, X is an αws-R_0 space.

Theorem 1.3:

The following two statements are equivalent

i) The topological space X is an αws-R_0 space.

ii) x ∈ αws-cl ({y}) iff y ∈ αws-cl ({x}) for any two points x and y in the topological space X.

Proof:

i)$\Longrightarrow$ ii)

Suppose X is an αws-R_0 space. Let $x \in$ αws-cl ({y}) and H be a αws-closed set containing x. Hence $x \in$ αws-cl ({y}) ∩ H. Assume that $y \notin H \Longrightarrow y \in X - H$. Since X is αws-$R_0$, by using Theorem 1.2, αws-cl ({y}) ∩ H = ϕ. This is a contradiction to the fact that the $x \in$ αws-cl ({y}) ∩ H. Therefore our assumption is wrong. Hence $y \in H$. Therefore, every αws-closed set containing x contains $y \Longrightarrow y \in$ αws-cl ({x}). Similarly if $y \in$ αws-cl ({x}), then we can prove that $x \in$ αws-cl ({y}).

ii) $\Longrightarrow$ i)

Suppose that $x \in$ αws-cl ({y}) iff $y \in$ αws-cl ({x}) for any two points x and y in X. To prove that X is an αws-R_0 space. Assume U is an αws-open set and $x \in U$. Take $y \in$ αws-cl ({x}). By our assumption $x \in$ αws-cl ({y}). If $y \notin U$, then $y \in X - U$. Since U is αws-open, $X - U$ is αws-closed and $x \notin X - U$. That is $x \notin$ αws-cl({y}), which is a contradiction to the fact that $x \in$ αws-cl ({y}). Hence $y \in U$ and αws-cl ({x}) ⊆ U. Then by using Definition 1.1, X is an αws-R_0 space.

Definition 1.4:

i) The intersection of all αws-open subsets of (X,τ) containing A is called αws-kernel of A (briefly, αws-ker(A)) That is, αws-ker(A) = ∩ { U : U ∈ αWSO(X, τ) and A ⊆ U}

ii) Let $x \in X$, then αws-kernel of x is denoted by αws-ker(x) = ∩ {U: U ∈ αWSO (X, τ) and x ∈ U}.

Proposition 1.5:

Let us consider X be a topological space and let A and B be subsets of X then

i) A ⊆ αws-ker(A)

ii) If A ⊆ B then αws-ker (A) ⊆ αws-ker(B)

iii) αws-ker (αws-ker(A)) = αws-ker(A) **iv)** If A is αws-open then A = αws-ker(A).

Proof:

Clearly (i) and (ii) follows from the definition of αws-kernel.

iii) By using (i) αws-ker(A) ⊆ αws-ker(αws-ker(A)). Let x ∈ αws-ker (αws-ker(A)).

Therefore every αws-open set containing αws-ker (A) contains x. Let U be an αws-open set containing A. Therefore αws-ker (A) ⊆ U and $x \in U$. Thus every αws-open set containing A contains x. Thus x ∈ αws-ker (A). Therefore, αws-ker (αws-ker (A)) ⊆ αws-ker (A). Hence αws-ker(A) = αws-ker (αws-ker (A)).

iv) Let A be any subset of X. We know that αws-ker (A) ⊆ A. By using (i), we get αws-ker (A) = A.

Proposition 1.6:

Let x and y be any two points in a space X. Then y ∈ αws-ker (x) iff x ∈ αws-cl({y})

Proof:

Let y ∈ αws-ker (A). To prove x ∈ αws-cl ({y}). If x ∉ αws-cl({y}), then there is an αws-closed set F such that y ∈ F and x ∉ F. Since F is an αws-closed set, X– F is an αws-open set containing x. Now, y ∉ ∩ {U: U ∈ αWSO(X, τ) and {x} ⊆ U} = αws-ker({x}), because y ∉ X − F. This is a contradiction, because y ∈ αws-ker({x}). Thus x ∈ αws-cl({y}). Conversely, suppose that x ∈ αws-cl({y}). To prove y ∈ αws-ker({x}). Suppose not, y ∉ αws-ker({x}) then there is an αws-open set U such that x ∈ U and y ∉ U ⟹ x ∉ X− U and X − U is an αws-closed set containing y. That is, x ∉ ∩ {F: F is αws-closed in X and {y} ⊆ F} = αws-cl({y}), which is a contradiction to the fact that x ∈ αws-cl({y}). Thereforey ∈ αws-ker({x}).

Theorem 1.7:

For any two points x, y in X then following two are equivalent

i) αws-ker({x}) ≠ αws-ker({y}) **ii)** αws-cl({x}) ≠ αws-cl({y})

Proof:

i) ⟹ ii)

Assume that αws-ker ({x}) ≠ αws-ker ({y}). Then there is a point z ∈ X such that z ∈ αws-ker ({x}) and z ∉ αws-ker ({y}) or z ∉ αws-ker ({x}) and z ∈ αws-ker ({y}). Consider the first case. we have x ∈ αws-cl ({z}) and y ∉ αws-cl ({z}). Since y ∉ αws-cl ({z}), there exists an αws-closed set H such that z ∈ H and y ∉ H. Since x ∈ αws-cl ({z}), x ∈ H. here y ∉ H and x ∈ H ⟹ y ∉ αws-cl ({x}). Obviously y ∈ αws-cl ({y}). Therefore αws-cl ({x}) ≠ αws-cl ({y}). Similarly we prove that second case.

ii) ⟹ i) Suppose that αws-cl ({x}) ≠ αws-cl ({y}). To prove αws-ker({x}) ≠ αws-ker ({y}). Then there is a point z ∈ X such that z ∈ αws-cl({x}) and z ∉ αws-cl({y}) or z ∉ αws-cl ({x}) and z ∈ αws-cl ({y}). Considering the first case. By using Proposition 1.6, we have x ∈ αws-ker ({z}) and y ∉ αws-ker ({z}). Since y ∉ αws-ker ({z}), there exists an αws-open set G such that z ∈ G and y ∉ G. Since x ∈ αws-ker ({z}), there exists an αws-open set G such that x ∈ G. Now, y ∉ G, x ∈ G ⟹ y ∉ αws-ker ({x}). Obviously y ∈ αws-ker ({y}). Hence αws-ker ({x}) ≠ αws-ker ({y}). Similarly we prove the second case.

Theorem 1.8:

A space X is an αws-R_0 space iff for any x ∈ X, αws-cl ({x}) = αws-ker ({x}).

Proof:

Let X be an αws-R_0 space. First we prove that αws-cl ({x}) ⊆ αws-ker ({x}). Let y ∈ αws-cl ({x}). If y ∉ αws-ker ({x}), then there exists an αws-open set U such that x ∈ U and y ∉ U. since U is an αws-open set, X− U is an αws-closed set. Since X is an αws-R_0 space, by using Theorem 1.2, αws-cl ({x}) ∩ (X− U) = ϕ. This is a contradiction to the fact that y ∈

αws-cl ({x}) ∩ (X− U) ⟹ y ∈ αws-ker ({x}). Thus αws-cl ({x}) ⊆ αws-ker ({x}). On the other hand, to prove αws-ker ({x}) ⊆ αws-cl({x}). Let y ∈ αws-ker ({x}). By using Proposition 1.6, x ∈ αws-cl ({y}). By using Theorem 1.3, y ∈ αws-cl ({x}). Thus αws-ker ({x}) ⊆ αws-cl ({x}). Therefore αws-cl ({x}) = αws-ker ({x}). Conversely, suppose that for any x ∈ X, αws-cl ({x}) = αws-ker ({x}). To prove that X is an αws-R_0 space. Let U be αws-open set in X such that x ∈ U. Therefore αws-cl ({x}) = αws-ker ({x}) = ∩ { G : G ∈ αWSO(X) and {x}⊆ G ⊆ U}. Then by using Definition 1.1, X is an αws-R_0 space.

Corollary 1.9:

If a space X is an αws-R_0 space, then for any αws-closed set H in X, αws-ker ({x}) ∩ H = ϕ for all x ∈ X – H.

Proof:

Suppose that X is an αws-R_0 space. Let H be an αws-closed set in X. By using Theorem 1.2, αws-cl ({x}) ∩ H = ϕ for all x ∈ X− H. By using Theorem 1.8, αws-ker ({x}) ∩ H = ϕ for all x ∈ X − H.

Theorem 1.10:

A space X is an αws-R_0 space iff for every αws-closed set H and x ∈ H, αws-ker({x}) ⊆ H.

Proof:

Let us assume that X be an αws-R_0 space. To prove that αws-ker ({x}) ⊆ H. Let H be any αws-closed set in X. Let x ∈ H. Thus αws-cl ({x}) ⊆ H. By using Theorem 1.8, αws-ker ({x}) ⊆ H. Conversely, assume αws-ker ({x}) ⊆ H, for every αws-closed set in H and x ∈ H. Let V be an αws-open set and x ∈ V. If y ∈ αws-cl ({x}), then by using Proposition 1.6, x ∈ αws-ker ({y}). If y ∉ V, then y ∈ X − V and X − V is αws-closed set. By using assumption, αws-ker({y}) ⊆ X − V. Since x ∈ αws-ker ({y}), x ∈ X − V and so x ∉ V, which is a contradiction to the fact that x ∈ V. Hence y ∈ V and αws-cl ({x}) ⊆ V. Then by using Definition 1.1, X is an αws-R_0 space.

Theorem 1.11:

A topological spaces X is an αws-R_0 space iff for any two points x, y ∈ X, αws-cl({x}) ≠ αws-cl({y}) implies αws-cl({x}) ∩ αws-cl({y}) = ϕ.

Proof:

Suppose X is an αws-R_0 space. To prove αws-cl ({x}) ∩ αws-cl({y}) = ϕ. Let us assume x, y ∈ X such that αws-cl({x}) ≠ αws-cl({y}). Then there exists an z ∈ X such that z ∈ αws-cl ({x}) and z ∉ αws-cl ({y}) or z ∉ αws-cl ({x}) and z ∈ αws-cl({y}). Considering the first case, then there exists an αws-closed set in F such that y ∈ F and z ∉ F. Now every αws-closed set containing x contains z, because z ∈ αws-cl ({x}). Thus x ∉ F and x ∈ X − F. Now, y ∈ F ⟹ αws-cl ({y}) ⊆ F. Hence αws-cl ({x}) ∩ αws-cl ({y}) ⊆ αws-cl ({x}) ∩ F = ϕ. By Theorem 1.2, the proof of the second case is similar to the first case. Conversely, suppose for any two points x, y ∈ X, αws-cl ({x}) ≠ αws-cl ({y}) ⟹ αws-cl ({x}) ∩ αws-cl ({y}) = ϕ. To prove X is an αws-R_0 space. Let V be an αws-open set and x ∈ V. Assume y ∈ αws-cl ({x}). If y ∉ V, then y ∈ X −V. Since x ∈ V, x ∉ X − V and X− V is αws-closed. Thus x ∉ αws-cl ({y}). Since x ∈ αws-cl ({x}), αws-cl ({x}) ≠ αws-cl ({y}). By our assumption, αws-cl ({x}) ∩ αws-cl ({y}) =ϕ. Thus y ∉ αws-cl ({x}) which is a contradiction to y ∈ αwscl({x}). Therefore, y ∈ V and αws-cl({x}) ⊆ V. Hence X is αws-R_0.

Theorem 1.12:

A topological spaces X is an αws-R_0 space iff for any two points x, y ∈ X, αws-ker ({x}) ≠ αws-ker ({y}) implies αws-ker ({x}) ∩ αws-ker ({y}) = ϕ.

Proof:

Given that Suppose X is an αws-R_0 space. To prove αws-ker({x}) ∩ αws-ker({y}) = ϕ. Let us assume x, y ∈ X such that αws-ker ({x}) ≠ αws-ker ({y}). By Theorem 1.7, αws-cl ({x}) ≠ αws-cl ({y}). By Theorem 1.10, αws-cl ({x}) ∩ αws-cl ({y}) = ϕ. Since X is αws-R_0, by Theorem 1.7, αws-ker ({x}) ∩ αws-ker ({y}) = ϕ. Conversely, assume that two points x, y ∈ X, αws-ker ({x}) ≠ αws-ker ({y}) implies αws-ker ({x}) ∩ αws-ker ({y}) = ϕ. To prove X is an αws-R_0 space. Let H be an αws-closed set and x ∈ H. Assume y ∈ αws-ker ({x}). If y ∉ H ⟹ y ∈ X− H and X − H is αws-open and x ∉ X− H. Hence x ∉ αws-ker ({y}). Since x ∈ αws-ker ({x}), αws-ker ({x}) ≠ αws-ker ({y}). By assumption, αws-ker ({x}) ∩ αws-ker ({y}) = ϕ. Thus y ∉ αws-ker ({x}), which is a contradiction to the fact that y ∈ αws-ker ({x}). Thus y ∈ H and αws-ker ({x}) ⊆ H. By Theorem 1.10, X is an αws-R_0 space.

Theorem 1.13:

A space X is αws-R_0 iff for each αws-closed set F and x ∉ F, there is an αws-open set U such that F ⊆ U, x ∉ U.

Proof:

Let us assume X be an αws-R_0 space and F be an αws-closed set in X that does not containing the point x. Therefore X − F is αws-open and x ∈ X− F. Since X is an αws-R_0, by using Definition 1.1, αws-cl ({x}) ⊆ X− F. This implies F ⊆ X − αws-cl ({x}). Let U= X − αws-cl ({x}). Therefore U is an αws-open set such that F ⊆ U and x ∉ U. Conversely, suppose that F is an αws-closed set and x ∉ F, there is an αws-open set U such that F ⊆ U, x ∉ U. Let us assume G be an αws-open set such that x ∈ G. Therefore X − G is an αws-closed set and x ∉ X − G. By using hypothesis, there exists an αws-open set U such that X − G ⊆ U and x ∉ U ⟹ X − U ⊆ G and x ∈ X− U. Thus αws-cl ({x}) ⊆ X − U ⊆ G. Then by using Definition 1.1, X is an αws-R_0 space.

Definition 1.14:

A topological space X is an **αws-R_1** space if for any x, y ∈ X with αws-cl ({x}) ≠ αws-cl ({y}), there exist disjoint αws-open sets U and V such that αws-cl ({x}) ⊆ U and αws-cl ({y}) ⊆ V.

Theorem 1.15:

If a space X is an αws-R_1 space, then for any x, y ∈ X with αws-ker ({x}) ≠ αws-ker ({y}), there exist disjoint αws-open sets U and V such that αws-ker ({x}) ⊆ U and αws-ker ({y}) ⊆ V.

Proof:

Assume that X is an αws-R_1 space. Let x, y ∈ X with αws-ker ({x}) ≠ αws-ker ({y}). By using Theorem 1.7, αws-cl ({x}) ≠ αws-cl ({y}). Since X is an αws-R_1 space, by using Definition 1.14, there exists disjoint αws-open sets U and V such that αws-cl ({x}) ⊆ U and αws-cl ({y}) ⊆ V. Since U and V are αws-open sets, αws-ker ({x}) ⊆ U and αws-ker ({y}) ⊆ V.

Proposition 1.16:

If X is αws-R_1, then it is an αws-R_0 space.

Proof:

Given that X is an αws-R_1 space. Let U be an αws-open set such that x ∈ U. suppose αws-cl ({x}) ⊄ U. There exists y ∈ X such that y ∈ αws-cl ({x}) and y ∉ U. Since x ∉ X− U and since X − U is an αws-closed set containing y, x ∉ αws-cl({y}). Therefore, αws-cl ({x}) ≠ αws-cl ({y}). Since X is an αws-R_1 space, by using Definition 1.14, there exists disjoint αws-open set G and V such that αws-cl ({x}) ⊆ G and αws-cl ({y}) ⊆ V and x ∉ V. Now, x ∈ X− V and X − V is an αws-closed and y ∉ X− V ⟹ y ∉ αws-cl({x}) which is a contradiction. Thus αws-cl({x}) ⊆ U. Therefore, X is an αws-R_1 space.

Theorem 1.17:

If a space X is both αws-T_0 and αws-R_0 then X is an αws-T_1.

Proof:

Suppose that X is both αws-T_0 and αws-R_0. Let x, y ∈ X. Since X is αws-T_0, there exists an αws-open set G such that x ∈ G and y ∉ G. Since X is αws-R_0, by using Definition 1.1, αws-cl ({x}) ⊆ G. Now y ∉ αws-ker ({x}), because y ∉ G. Since X is αws-R_0, by using Theorem 1.8, y ∉ αws-cl ({x}). Hence y ∈ X − αws-cl ({x}) and x ∉ X− αws-cl ({x}). Suppose take H = X − αws-cl ({x}). Thus y ∈ H and x ∉ H. Also H is αws-open in X. Hence there exists disjoint αws-open sets G and H such that x ∈ G but y ∉ G and x ∉ H but y ∈ H. Hence X is an αws-T_1 space.

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