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Group's Uncertainty Defuzzification Method To Select The Best Four Wheeler Group For Public Transport By Private Or Government Sector

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Abstract: -
In this article, we deal with many types of vehicles and their parameters using fuzzy logic as tool and also apply fuzzy rule to achieve this objective. A rule can be generated by five parameters as journey time factor, journey cost, facility, sitting arrangement and satisfaction of the vehicles at journey time for public. In this article, we develop a fuzzy model for assessing vehicle's group's parameter using time at public transportation. These parameters under assessment are represented as a fuzzy subset of the set of linguistic label's and their performance according to the vehicles groups. And the possibilities of all vehicles profile are calculated using group's uncertainty defuzzification method.

Key words: Fuzzy Logic, Fuzzy set, Defuzzification method, the group's Uncertainty method.

1. Introduction

Number of private vehicles in India are growing very rapidly. Poor public transportation system in most of the urban cities has enforced many middle class to switch to own a vehicle for their daily life. This paper is an attempt to choose suitable vehicles group according to their performance for public transport by private or government sector. Vehicles are very important and complex system for public transport which transports people from one place to another place. Fuzzy Logic, which is based on fuzzy set theory introduced by Zadeh in 1965 **[12].** Fuzzy set theory allows an object belonging to multiple exclusive set in the universal set and each set has its own membership function which defined or determinate the degree of truth that an element belong to the set. Fuzzy set theory proposed in terms of membership function operating over the range [0, 1] of real number **[12, 13]**. Fuzzy set theory is distinct from probability theory for example: the probabilitistic theory yield natural language statement "there is a 70% chance that Rita is short" while according to the fuzzy rule, the statement "Rita's degree of membership within the set of short people is 0.70%". The semantic difference is significant that the first statement show that Rita is or is not short in which set Rita is in, but according to fuzzy logic suppose that Rita is more or less short or some other terms corresponding to the value of 0.70% **[14]**. Fuzzy logic is most problem solving method than to probability theory. In survey, such situation often appear in the case of assessment the five parameter's of vehicle of modeling and etc. According to survey of vehicles groups, the passengers points of views, there usually exist vagueness about the degree of performance of vehicles in each stage of the corresponding situation **[8, 9, 10]**.

2. Definition and Preliminaries.

2.1 [3] Possibility Distribution:- 12

Let possibility distribution function r be defined on X = {x_1, x_2, …. , x_n}. Possibility distribution associated with r is n-tuple **r** = (r_1, r_2, …, r_n) where $r_i = r(x_i)$ for all $x_i \in X$. It is useful to order **r** such that $r_i \geq r_j$ when i < j.

2.2 [1, 3] Basic Distribution:-

Every possibility distribution **r** on finite set can be represented by n-tuple **m** = (m_1, m_2, …, m_n) for some finite n ∈ N_n where $m_i = m(A_i)$ for all i ∈ N_n. Clearly, P_{n_i} =1, m_i = 1 and $m_i \in [0, 1]$ for all i ∈ I_n n, m is called basic distribution, each m represents exactly one possibility distribution **r**

$$\forall \quad x_i \in X: \quad r_i = r(x_i) = Pos(\{x_i\}) = PI(\{x_i\})$$

$$\forall \quad i \in N_n: \quad r_i = PI(\{x_i\}) = \sum_{k=i}^{n} m(A_k) = \sum_{k=i}^{n} m_k .$$

Solving all n equations for m_i ($i \in N_n$) leads to $m_i = r_i - r_{i+1}$, one-two-one correspondence between **r** and **m.**

2.3 [2, 3] Probability p(s):-

The probability of the parameters is defined in a way analogous to crisp data i.e., by

$p(s) = \frac{ms}{\sum m\, s}$ and it is denoted by p(s).

2.4 [3] Possibility r(s):-

The possibility of s to be $r(s) = \frac{ms}{\max\{ms\}}$ where max {m_s} denotes the maximal value of m_s for all s in U_3. In other words the possibility of s expresses the "relative membership degree" of s with respect to max {m}.

3. Methodology

Let us consider a class of n vehicles n ≥1 and let us assume that according to the survey, we want to assess the following vehicles as train S_1, car S_2 and bus S_3 and their parameters chosen for evaluate our model denoted by a, b, c, d and e, defined as the linguistic label (fuzzy expression) as journey time factor, journey cost, facility, sitting arrangement and satisfaction respectively of vehicles in each of the S_i and set U = {a, b, c, d, e} where S_i, i = 1, 2, 3 a fuzzy subset A_i of U and U is a universal set of X.

Now we want to apply probabilistic standards in assuring the judgement of the vehicles performance at each stage of the parameters, then we should use the relative frequencies n_{ix} / n.

According to fuzzy set and its membership function m_{Ai} (x) is usually defined in terms of Logical or / and statistical data the fuzzy subset A_i of U corresponding to S_i has the form: $A_i = \{(x, m_{Ai}(x)): x \in U\}$, i = 1, 2, 3 all possible vehicles profiles with respect to the assessing Process. We consider a fuzzy relation is denoted R in U_3 (i.e., a fuzzy subset of U_3) of the form: R = {(s, $m_{R(s)}$): s = (x, y, z) U_3} where for determining properly the membership function mR we give the following definition: A profile s = (x, y, z), with x, y, z in U, is said to be well ordered if x corresponds to degree of performance equal or greater than y and y corresponds to a degree of performance equal or greater than z. For example, (a, b, b) is a well ordered profile, while (b, a, c) is not.
We define now the membership degree of a profile s to be $m_R(s) = m_{A_1}(x), m_{A_2}(y), m_{A_3}(z)$, if s is well ordered and 0 otherwise. We shall write already m_s instead of $m_R(s)$. We considered from the above two definitions the probability and possibility it becomes evident that p(s) < r(s) for all s in U_3, which is compatible to the common logic. In fact, whatever is probable it is also possible?
In this paper let us consider probability and possibility this two definitions wants to study the combined results of the performance of k different groups of vehicles k ≥2. For this, we introduce the fuzzy variables $A_1(t)$, $A_2(t)$ and $A_3(t)$ with t = 1, 2, …, k. The values of these variables represent fuzzy subsets of U corresponding to the vehicles parameters under assessment for each of the groups Obviously, in order to measure the degree of evidence of the combined results of the k groups, it is necessary to define the probability p(s) and the possibility r(s) of each profile s with respect to the membership degrees of s for all groups. For this reason we introduce the pseudo-frequencies f(s) = $m_s(1) + m_s(2) + \dots + m_s(k)$ and we define the probability and possibility of a profile s by p(s) = $\frac{f(s)}{\Sigma f(s)}$ and r(s) = $\frac{f(s)}{\max\{f(s)\}}$ respectively, where max{f(s)} denotes the maximal pseudo-frequency **[2, 3]**. The same method could be applied for the second group. The above model gives, through the calculation of probabilities and possibilities of all vehicles profiles, a quantitative/qualitative view of their realistic performance.

3.1. The Group's Uncertainty Method :

We known that the collection of data obtained by a performance can be measured by the reduction of uncertainty resulting from this action accordingly a system's situation of uncertainty. Therefore, a measure of uncertainty could be adopted as an alternative deffuzification technique for the vehicles groups' assessment model developed in the above section within the domain of possibility theory uncertainty consists of strife (or discord), which expresses conflicts among the various sets of alternatives, and non-specificity (or imprecision), which indicates that some alternatives are left unspecified, i.e., it expresses conflicts among the sizes (cardinalities) of the various sets of alternatives **[4, 5]**. Strife is measured by the function ST(r) on the ordered possibility distribution r: $r_1 = 1$ □□r_2 □□……. □□r_m □□r_{m+1} of a group of vehicles defined by

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$$ST(r) = 1/\log 2 \; [\sum_{i=2} [(r_i - r_{i+j}) \log i / \sum_{j=1} r_j]]$$

Similarly, non-specificity is measured by the function N(r) = 1

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$$N(r) = 1/\log 2 \; [\sum_{i=2} [(r_i - r_{i+j}) \log i]$$

The sum T(r) = ST(r) + N(r) is a measure of the total possibility uncertainty for ordered possibility distributions. The lower is the value of T(r), which means greater reduction of the initially existing uncertainty, the better the system's performance. We must emphasize that the deffuzification methods presented above treat differently the idea of a group's performance **[6, 9]**. Now let us consider an example:

4. Numerical Examples

The following data was obtained by assessing the mathematical skills of two groups of different type of four wheelers Vehicles the groups of cars, the groups of buses and train in private and Government sector for public transport. This survey starts from INDORE to nearest cities as UJJAIN and INDORE to nearest cities as MHOW by BHAWNA. here we discuss first group obtained four wheeler as 10 cars , 5 buses and 1 train from Indore to Ujjain judge according to survey its features with membership function is given below:

A_{11} = {(a, 0), (b, 0.25), (c, 0.5), (d, 0.25), (e, 0.25)},
A_{12} = {(a, 0.25), (b, 0), (c, 0.25), (d, 0), (e, 0.5)},
A_{13} = {(a, 0), (b, 0.25), (c, 0.25), (d, 0.5), (e, 0)}

According to the above notation the first index of A_{ij} denotes the group (i = 1, 2) and the second Index denotes the corresponding vehicles characteristic Sj (j = 1, 2, 3). Now we calculated the membership degrees of the U3 in total possible Vehicles' profiles as it is described in Section 3.1 (see column of ms (1) in Table 1). For example, for the profile s = (c, e, d)) find the membership degree $m_s = 0.5 \times 0.5 \times 0.25 = 0.06225$. From the values of the column of m_s (1) it turns out that the maximal membership degree of vehicles profiles is 0.06225 **[7, 15]**. Therefore, the Possibility of each s in U_3 is given by $r_s = 0.06225$. The possibilities of the vehicles profiles are presented in column of $r(s_1)$ of Table 1. One could also calculate the probabilities of the vehicles Profiles using the formula for p(s) given in section 3.

Table 1. Profiles with non-zero membership degrees.

A_1	A_2	A_3	$m(s_1)$	$r(s_1)$	$m(s_2)$	$r(s_2)$	f(s)	r(s)

b	a	b	0.0156	0.249	0.0625	1	0.0781	0.0624
b	a	c	0.0156	0.249	0.0625	1	0.0781	0.0624
b	b	c	0	0	0	0	0	0
b	c	a	0	0	0	0	0	0
b	c	b	0.0156	0.249	0.031	0.496	0.0466	0.273
b	c	c	0.0156	0.249	0.0156	0.249	0.0312	0.249
b	c	d	0.031	0.496	0	0	0.031	0.248
b	c	e	0	0	0	0	0	0
b	d	e	0	0	0	0	0	0
c	a	b	0.031	0.496	0.031	0.496	0.062	0.496
c	c	c	0.031	0.496	0.0156	0.249	0.0466	0.273
c	c	d	0.0625	1	0	0	0.0625	0.5
c	d	e	0	0	0	0	0	0
c	e	d	0.031	0.496	0	0	0.031	0.248
d	a	c	0	0	0.031	0.496	0	0.248
d	e	a	0	0	0.0625	1	0	0.5
d	e	e	0	0	0	0	0	0
e	a	b	0.0156	0.249	0	0	0.0156	0.124
e	c	a	0	0	0	0	0	0
e	c	c	0.0156	0.249	0	0	0.0156	0.124
e	c	d	0.031	0.496	0	0	0.031	0.248
e	e	a	0	0	0	0	0	0
e	e	b	0.031	0.496	0	0	0.031	0.248
e	e	c	0.031	0.496	0	0	0.031	0.248
e	e	d	0.0625	1	0	0	0.0625	0.5

The outcomes of Table 1 are with accuracy up to the third decimal point.
Similarly survey on vehicle's in second group as INDORE to MHOW data prepared by me. Let as consider an example:

A_{21} = {(a, 0), (b, 0.5), (c, 0.25), (d, 0.25), (e, 0)},
A_{22}= {(a, 0.5), (b, 0), (c, 0.25), (d, 0), (e, 0.25)}
A_{23} = {(a, 0.25), (b, 0.25), (c, 0.25), (d, 0), (e, 0)}.

The membership degrees and the possibilities of profile vehicles are presented in columns of $m_s(2)$ and $r_s(2)$ of Table 1 respectively . In order to study the combined results of the two groups' performance we also calculated the pseudo-frequencies $f(s) = m_s(1) + m_s(2)$ and the combined possibilities of all profiles presented in the last two columns of Table 1.We compare now the two groups' performance by applying this method. Calculating the possibilities of all profiles (column of $r(s_1)$ in Table 1) one finds that the ordered possibility distribution for the first vehicles group is:

$r: r_1 = r_2 = r_3 = 1, r_4 = r_5 = r_6 = r_7 = r_8 = 0.496, r_9 = r_{10} = 0.249, r_{11} = r_{12} = r_{13} = r_{14} = r_{15} = r_{16} = 0.124$, $r_{17} \ldots\ldots = r_{125} = 0$.

Thus, with the help of a calculator one finds that

$$ST(r) = 1/\log 2 \left[\sum_{i=2}^{14} \left[(r_i - r_{i+j}) \log i / \sum_{j=1}^{i} r_j\right]\right]$$

$$ST(r) = 1/0.301 [(0.504\log 3/ 3 + 0.247 \log 8/ 5.048 + 0.125 \log 10/ 5.978 + 0.124 \log 16/ 6.722)]$$

$$ST(r) = 3.32 (0.504\times 0 + 0.247\times 0.164 + 0.125\times 0.223 + 0.124\times 0.3766)$$

$$ST(r) = 3.32 (0+0.0405 + 0.0279 + 0.0105)$$

$$ST(r) = 3.32\times 0.0789$$

$$ST(r) = 0.261$$

Similarly

$$N(r) = 1/\log 2 \left[\sum_{i=2}^{14} [(r_i - r_{i+j}) \log i]\right]$$

$$N(r) = 1/0.301[(0.504\log 3+0.247 \log 8+0.125 \log 10+0.124 \log 16)]$$

$$N(r) = 3.32 (0.504\times 0.4771 + 0.247\times 0.9030 + 0.125\times 1 + 0.124\times 1.204)$$

$$N(r) = 3.32 (0.240+0.223+0.125+0.1493)$$

$$N(r) = 3.32\times 0.7373,$$

$$N(r) = 2.4478$$

The sum $T(r) = ST(r) + N(r)$

$$T(r) = 0.261+2.447 = 2.7088$$

The ordered possibility distribution for the second vehicles group (column of $r_s(2)$ in Table 1) is:

$r: r_1 = r_2 = r_3 = 1, r_4 = r_5 = r_6 = 0.496, r_7 = r_8 = 0.249, r_9 = r_{10} = r_{11} = r_{12} = r_{13} = r_{14} = r_{15} = \ldots\ldots = r_{125} = 0$

$$ST(r) = 1/\log 2 \left[\sum_{i=2}^{14} [(r_i - r_{i+j}) \log i / \sum_{j=1}^{i} r_j]\right]$$

$$ST(r) = 1/0.301\ (0.504\log 3/ 3 + 0.247 \log 6/ 4.488 + 0.249 \log 8/ 4.986)$$

$$ST(r) = 3.32\ (0.504\times 0 + 0.247\times 0.126 + 0.249\times 0.205)$$

$$ST(r) = 3.32\ (0+0.0311 + 0.05112)$$

$$ST(r) = 3.32\times 0.08222$$

$$ST(r) = 0.273000$$

Similarly

$$N(r) = 1/\log 2 \left[\sum_{i=2}^{14} [(r_i - r_{i+j}) \log i]\right]$$

$$N(r) = 1/0.301\ [(0.504\log 3+0.247 \log 6+0.125 \log 8)]$$

$$N(r) = 3.32\ (0.504\times 0.4771 + 0.247\times 0.7781 + 0.249\times 0.9030)$$

$$N(r) = 3.32\ (0.240+0.192+0.224)$$

$$N(r) = 3.32\times 0.6565$$

$$N(r) = 2.1798$$

The sum find that $T(r) = ST(r) + N(r)$

$$T(r) = 0.273000+2.1798 = 2.45288$$

Therefore, since, $2.7088 > 2.45288$ it turns out that the second in general a slightly better average performance than the first one.

5. Result

Clearly the group's uncertainty defuzzification techniques method gives the result of the vehicle's and their features assessment. According to this method, we find the value of probability p(s) and the possibility r(s) then find ST(r) , N(r) and lastly find that T(r) = ST(r) + N(r). With this method, we easily compare two vehicle's group. In this article, we observed that the first group's performance is better by private sector for public transport than the second one.

6. Conclusions

The first-order vehicle transfer function that has been utilized in this paper depends on the fuzzy model. The Group's Uncertainty method gives the better results for the selection of best four wheeler vehicles groups using public transport based on their characteristics by the recruiters. Three types of four wheeler vehicle starting up from journey time to satisfaction by using the FLC model has been explained. We can easily calculate vehicle assessment numerical value using this fuzzy logic defuzzification technique and compare both groups then find that first group has a better performance in public transport.

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