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Fuzzy Soft Sets (Fsss) Distance And Similarity Measures

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Abstract

In this paper, variety of new distance and similarity measures is proposed for Fuzzy Soft Sets. The Fuzzy Soft Sets (FSSs) is a powerful mathematical tool for decision making problem. Therefore the Fuzzy Soft distance and similarity measures are proposed to solve the multiple criteria decision making problems. The different distance and similarity measures are based on Hamming distance, Euclidean distance, Hausdorff metric, normalized Hamming distance, normalized Euclidean distance, normalized Hausdorff distance, generalized Hamming distance, generalized Euclidean distance and generalized Hausdorff distance. An application of Fuzzy Soft distance and similarity measures is illustrated for decision making problem.

Keywords

Fuzzy sets, Soft sets, Fuzzy Soft sets, Fuzzy Soft Distance, Fuzzy Soft Similarity.

1. Introduction

The real world has uncertainty, imprecision and vagueness. Many problems in the domains such as bio-informatics, environment, medical science, economics, engineering and social science involve uncertain, imprecise, fuzzy and not clearly defined data. Recently several researchers have worked in modeling vagueness. The traditional mathematical approaches are not always successful to solve these problems. L.A. Zadeh proposed fuzzy set theory [1] as an extension of classical set theory. The applications of fuzzy set theory have been widely used in the various areas like: System control, aircraft control, data analysis and pattern recognition. Fuzzy approach is commonly used to deal with imprecise and vague in decision making problems Fuzzy set theory became a powerful tool to handle uncertain and imprecise data. While many existing theories such as probability theory, rough set [2], intuitionist fuzzy sets [15]. Vague sets and interval mathematics [1] have been developed to model vagueness. However, these approaches have inherent difficulties as pointed out in [3]. Molodtsov [3,7] initiated the concept of soft set theory as a new mathematical tool to deal with uncertainties. Many researchers have done their research to do theoretic study of soft set theory [1-6,9]. The soft set theory is free from the aforementioned limitations. It has no problem of setting the membership function, which makes it very convenient and easy to apply in practice [3, 6].Pawlak [2](1982) introduced the theory of Rough sets.

P.K. Maji et al.[4,5] proposed the fuzzy soft sets concepts and used the applications in decision making problems. P.K. Maji[6] redefined the soft set theory (2003). D Chen et al.[8] presented the parameterization reduction concepts of soft sets and its applications. H Aktas and N Cagman [9] proposed the soft sets and soft groups and introduced some properties. In 2007 A.R. Roy [10] introduced a fuzzy soft set theoretic approach for decision making problems. Hong-Ying Zhang et al.[11] introduced hybrid monotonic inclusion measure and its use for measuring similarity and distance between fuzzy sets. Debashree Guha et al.[12] presented a new approach to fuzzy distance measure and similarity measure between two generalized fuzzy numbers. M Adabitabar Firozja et al.[13] proposed the similarity measures of generalized fuzzy numbers based on interval distance. G.A. Papakostas et al.[14] described new distance and similarity measures between intuitionistic fuzzy sets. And a comparative analysis has been done from a pattern recognition point of view. Yuncheng Jiang et al.[15] introduced an entropy on intuitionistic fuzzy soft sets and on interval-valued fuzzy soft sets. Chang Wang et al.[16] presented entropy, similarity measure and distance measure of vague soft sets and their relations. Ding-Hong Peng et al.[17] proposed generalized hesitant fuzzy synergetic weighted distance measures and their application to multiple criteria decision-making. Huimin Zhang et al.[18] presented some distance measures between intuitionistic fuzzy sets and interval-valued fuzzy sets. Wei Yang[19] defined new similarity measures for soft sets and their application. Inés Couso et al.[20] introduced the similarity and dissimilarity measures between fuzzy sets. It is a formal relational study based on similarity and dissimilarity measures fuzzy sets. Huchang Liao et al.[21] proposed distance and similarity measures for hesitant fuzzy linguistic term sets and their application in multi-criteria decision making. B Farhadinia [22] presented some distance and similarity measures for higher order hesitant fuzzy sets. Naim C et al.[23] introduced the similarity measures of intuitionistic fuzzy soft sets and their decision making. Qinrong Feng et al.[24] proposed the new similarity measures of fuzzy soft sets based on distance measures. Anjan Mukherjee et al.[25] described the similarity measures of interval-valued fuzzy soft sets and their application in decision making problems. Zeshui Xu et al.[26] presented distance and similarity measures for hesitant fuzzy sets. Huchang Liao et al.[27] introduced distance and similarity measures for hesitant fuzzy linguistic term sets. Their applications are multi-criteria decision making [28] [29]. Mikael Collan et al. [30] proposed new fuzzy similarity measure based TOPSIS variants for evaluating R&D projects as investments and overall ranking. G S Thakur [31] proposed the applications of Fuzzy Soft for Traffic Accident Alert Model.
The aforementioned literature shows different measures, however, cannot be used to deal with the distance and similarity between Fuzzy Soft Sets (FSSs).Due to the fact that Fuzzy Soft Sets is a very powerful mathematical tool to deal with uncertainties and vagueness in decision making as mentioned earlier, it is necessary to develop some measures for FSSs. This paper is organized as follows. In Section 2,we present the preliminaries and basic definition. In Section 3, we present the proposed Fuzzy Soft distance and similarities measures. Section 4 presents an application in decision making problem. Finally, the concluding remarks and future works are discussed in Section 5.

2. Preliminaries and Basic Definitions

Soft sets

The soft set theory [1-6], proposed by Russian researcher Molodtsov[3,7], in 1999 as a new generic mathematical tool for dealing with the uncertain data.

Definition 1([3,7]). Let U be an initial universe set and E be a set of parameters. Let P (U) denotes the power set of U and A⊂ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→P (U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For ε∈ A, F (ε) may be considered as the set of ε-approximate elements of the soft set (F, A).

Example 1: Suppose that Mr. X wants to buy a house. Let U={h_1,h_2,h_3,h_4,h_5}be a set of houses under consideration. Let A= {e_1, e_2, e_3} be a set of parameters where e_1=expensive,e_2=beautiful and e_3=in the good location. Suppose that

$F (e_1) = \{h_1, h_5\}$
$F (e_2) = \{h_1, h_3, h_5\}$
$F (e_3) = \{h_1, h_4\}$

The soft set (F, A) is describe the "attractiveness of the houses". F(e_1) means "house(expensive)" whose function-value is the set { h_1,h_5}, F(e_2) means "house(beautiful)" whose function-value is the set { h_1,h_3,h_5}and F(e_3) means "house(in the good location)" whose function-value is the set { h_1, h_4}.

Fuzzy soft sets

The real world is inherently uncertain, imprecise and vague. Traditional mathematical tools cannot deal with such problems. The fuzzy sets theory is widely employed in such kinds of problems. Maji and Biswas et al. [7,9] proposed the notion of fuzzy soft sets and an example of decision-making[5] discussed.

Definition 2. [4,5]: Let U be an initial universe set and E be a set of parameters. Let P (U) denotes the set of all fuzzy sets of U and A ⊂ E. A pair (F,A) is called a fuzzy soft set over U, where F is a mapping given by F: A→P (U).

Example 2**:** Consider example 1.In real life,in fact,much information is fuzzy,we can't describe fuzzy information with only two numbers 0 and 1, we often use a membership function instead of the crisp number 0 and 1 to characterize it.Then the fuzzy soft set (F,A) can describe the "attractiveness of the houses" under the Fuzzy circumstances.

$F(e_1) = \{ h_1 /0.5, h_2/0.7, h_3 /0.6,h_4/0.8, h_5/0.3\}$
$F(e_2) = \{ h_1 /0.9, h_2/0.4, h_3 /0.8, h_4/0.3, h_5/0.2\}$
$F(e_3) = \{ h_1 /0.5, h_2/0.4,h_3 /0.8, h_4/0.5, h_5/0.8\}$

3. Proposed Fuzzy Soft Distance Measures

Definition 3. (Fuzzy Soft Distance Measures [24]) Let (F,M) and (G,N) be two fuzzy soft sets over (U,E). The Fuzzy Soft Distance Measures between satisfy the following properties

1. $0 \le d((F,M),(G,N)) \le 1$;
2. $d((F,M),(G,N)) = 0$, if $(F,M) = (G,N)$;
3. $d((F,M),(G,N)) = d((G,M),(F,N))$;

Let (H,C) be a fuzzy soft set, if $(F,M) \sim \subseteq (G,N) \sim \subseteq (H,C)$, then $d((F,M),(G,N)) \le d((F,M),(H,C))$ and $d((G,N),(H,C)) \le d((F,M),(H, C))$.

Definition 4. (Fuzzy Soft Similarity Measures [24]) Let (F,A) and (G,B) be two fuzzy soft sets over (U,E). Then similarity measure between (F,A) and (G,B) is defined as $\xi((F,A),(G,B))$ i.e.

$\xi_i((F,A),(G,B)) = 1 - d_i((F,A),(G,B))$

Where i=1 to 40

$\xi((F,A),(G,B))$ Satisfies following properties.

(a) $0 \le \xi((F,A),(G,B)) \le 1$,

(b) $\xi((F,A),(G,B)) = 1$, if (F,A) =(G,B),

(c) $\xi((F,A),(G,B)) = \xi((G,B),(F,A))$

Let (H, C) be a fuzzy soft set, if (F, A) ~ ⊆(G,B) ~ ⊆(H,C),then

$\xi((F,A),(H,C)) \le \xi((F,A),(G,B))$ *and* $\xi((F,A),(H,C)) \le \xi((G,B),(H,C))$.

The distance d_1 between (x_i, y_i) and (x'_i, y'_i) is defined as follows:

$$d_1 = \sum_{i=1}^{n} \left|x_i - x'_i\right|^2 + \left|y_i - y'_i\right|^2$$

The Fuzzy Soft Distance Measures between (F,M) and (G,N) are defined as follows:

The fuzzy soft Hamming distance (d_2) is given as:

$$d_2((F,M),(G,N)) = \sum_{i=1}^{m} \sum_{j=1}^{n} |F(ei)(xj) - G(ei)(xj)|;$$

The fuzzy soft Euclidean distance (d_3) is given as:

$$d_3((F,M),(G,N)) = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} |F(ei)(xj) - G(ei)(xj)|^2\right)^{1/2};$$

The fuzzy soft normalized Euclidean distance (d_4) is given as:

$$d_4((F,M),(G,N)) = \frac{1}{n}\left(\sum_{i=1}^{m} \sum_{j=1}^{n} |F(ei)(xj) - G(ei)(xj)|^2\right)^{1/2};$$

The fuzzy soft Haudorff metric distance(d_5) is given as:

$$d_5((F,M),(G,N)) = \max |F(ei)(xj) - G(ei)(xj)|;$$

The fuzzy soft normalized Hamming distance(d_6) is defined as follows :

$$d_6((F,M),(G,N)) = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{lxi}\sum_{j=1}^{lxi} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|\right];$$

The Hausdorff metric is applied to the distance measure,then a generalized fuzzy soft normalized Hausd orff distance(d_7) is defined as

$$d_7((F,M),(G,N)) = \left[\frac{1}{n}\sum_{i=1}^{n} \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda}\right]^{1/\lambda};$$

Where λ>0.

From generalized fuzzy soft normalized Hausdorff distance (d_7),we have two special cases of the distance d_7:

Case 1: If λ = 1, then d_7 becomes a fuzzy soft normalized Hamming Hausdorff distance (d_8):

$$d_8((F,M),(G,N)) = \frac{1}{n}\sum_{i=1}^{n} \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|;$$

Case 2: If λ = 2, then d_7 becomes a fuzzy soft normalized Euclidean Hausdorff distance (d_9):

$$d_9((F,M),(G,N)) = \left[\frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{lxi}\sum_{j=1}^{lxi} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2\right]\right]^{1/2}$$

If we merged the above distances d_8 and d_9, then a generalized hybrid fuzzy soft normalized distance (d_{12}), hybrid fuzzy soft normalized Euclidean distance (d_{11}) and a hybrid fuzzy soft normalized Hamming distance (d_{10}) are as follows:

$$d_{10}((F,M),(G,N)) = \left[\frac{1}{n}\sum_{i=1}^{n}\left[\frac{1}{lxi}\sum_{j=1}^{lxi} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda}\right]\right]^{1/\lambda}$$

$$d_{11}((F,M),(G,N)) = \left[\left[\frac{1}{n}\sum_{i=1}^{n} \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2\right]\right]^{1/2}$$

$$d_{12}((F,M),(G,N)) = \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{1}{lxi}\sum_{j=1}^{lxi} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)| + \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|\right)$$

$$d_{13}((F,M),(G,N)) = \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{1}{lxi}\sum_{j=1}^{lxi} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2 + \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2\right)^{1/2}$$

$$d_{14}((F,M),(G,N)) = \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{1}{lxi}\sum_{j=1}^{lxi} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda} + \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda}\right)^{1/\lambda}$$

Where λ>0.

This section presents the following weighted distance measures for fuzzy soft sets. The generalized fuzzy soft weighted distance (d_{15}) is defined as, when the element of X is $w_i \in [0,1]$ and $\sum_{i=1}^{n} wi = 1$,

$$d_{15}((F,M),(G,N)) = \left[\sum_{i=1}^{n} wi\left(\frac{1}{lxi}\sum_{j=1}^{lxi} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda}\right)\right]^{1/\lambda}$$

And a generalized fuzzy soft weighted Hausdorff distance (d_{16}):

$$d_{16}((F,M),(G,N)) = \left[\sum_{i=1}^{n} wi \max_j \left(|F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda}\right)\right]^{1/\lambda}$$

Where λ > 0.

Now we discuss two special cases:

Case 1: if λ = 1, then we obtained a fuzzy soft weighted Hamming distance(d_{17}):

$$d_{17}((F,M),(G,N)) = \sum_{i=1}^{n} wi\left[\frac{1}{lxi}\sum_{j=1}^{lxi} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|\right]$$

And a fuzzy soft weighted Hamming–Hausdorff distance (d_{18}):

$$d_{18}((F,M),(G,N)) = \sum_{i=1}^{n} wi \max_j \mid F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj) \mid$$

Case 2: if λ = 2, then a fuzzy soft weighted Euclidean distance (d_{19}):

$$d_{19}((F,M),(G,N)) = \left[\sum_{i=1}^{n} wi \left(\frac{1}{lxi}\sum_{j=1}^{lxi} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2\right)\right]^{1/2}$$

And a fuzzy soft weighted Euclidean–Haudorff distance (d_{20}):

$$d_{20}((F,M),(G,N)) = \left[\sum_{i=1}^{n} wi \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2\right]^{1/2}$$

Furthermore, a generalized hybrid fuzzy soft weighted distance (d_{21}) is developed after combining the generalized fuzzy soft weighted distance and the generalized fuzzy soft weighted Hausdorff distance as:

$$d_{21}((F,M),(G,N)) = \left[\sum_{i=1}^{n} wi \left(\frac{1}{lxi}\sum_{j=1}^{lxi} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda} + \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda}\right)\right]^{\lambda}$$

Where λ >0.

From a generalized hybrid fuzzy soft weighted distance (d_{21}), we can derive another two distances d_{22}, d_{23}. If λ = 1, 2, then d_{21} is produced a hybrid fuzzy soft weighted Hamming distance (d_{22}) and a hybrid fuzzy soft weighted Euclidean distance (d_{23}) as follows:

$$d_{22}((F,M),(G,N)) = \sum_{i=1}^{n} wi \left(\frac{1}{lxi}\sum_{j=1}^{lxi} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)| + \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|\right)$$ $$d_{23}((F,M),(G,N)) = \left[\sum_{i=1}^{n} wi \left(\frac{1}{lxi}\sum_{j=1}^{lxi} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2 + \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2\right)\right]^{1/2}$$

All the aforementioned fuzzy soft distance measures are discrete. In this section, we proposed continuous fuzzy soft distance measures. If $x \in X = [a,b]$ is w_i and $\int_a^b w(x)dx = 1$, then a continuous fuzzy soft weighted Hamming distance(d_{24}), a continuous fuzzy soft weighted Euclidean distance(d_{25}) and a generalized continuous fuzzy soft weighted distance(d_{26}) are defined as follows, respectively:

$$d_{24}((F,M),(G,N)) = \int_a^b w(x)\left(\frac{1}{lx}\sum_{j=1}^{lx} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|\right)dx$$ $$d_{25}((F,M),(G,N)) = \left[\int_a^b w(x)\left(\frac{1}{lx}\sum_{j=1}^{lx} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2\right)dx\right]^{1/2}$$

$$d_{26}((F,M),(G,N)) = \left[\int_a^b w(x)\left(\frac{1}{lx}\sum_{j=1}^{lx} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda}\right)dx\right]^{1/\lambda}$$

Where λ >0.

A continuous fuzzy soft normalized Hamming distance (d_{27}) is derived from the continuous fuzzy soft weighted Hamming distance(d_{24}).If $w(x)=\frac{1}{(b-a)}$ for all $x \in [a,b]$, then we get (d_{27}):

$$d_{27}((F,M),(G,N)) = \frac{1}{b-a}\int_a^b \left(\frac{1}{lx}\sum_{j=1}^{lx} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|\right)dx$$

While the continuous fuzzy soft weighted Euclidean distance (d_{25}) is derived a continuous fuzzy soft normalized Euclidean distance (d_{28}):

$$d_{28}((F,M),(G,N)) = \left[\frac{1}{b-a}\int_a^b \left(\frac{1}{lx}\sum_{j=1}^{lx} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2 \right) dx \right]^{1/2}$$

And the generalized continuous fuzzy soft weighted distance (d_{26}) is derived a generalized continuous fuzzy soft normalized distance (d_{29}):

$$d_{29}((F,M),(G,N)) = \left[\frac{1}{b-a}\int_a^b \left(\frac{1}{lx}\sum_{j=1}^{lx} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda} \right) dx \right]^{1/\lambda}$$

Where $\lambda > 0$.

The traditional Hausdorff metric is derived generalized continuous fuzzy soft weighted Hausdorff distance (d_{30}) as follows:

$$d_{30}((F,M),(G,N)) = \left[\int_a^b w(x) \max j \left(|F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda} \right) dx \right]^{1/\lambda}$$

Where $\lambda > 0$.

From the generalized continuous fuzzy soft weighted Hausdorff distance (d_{30}),we have two special cases if $\lambda = 1, 2$, then distance (d_{30}) is produced a continuous fuzzy soft weighted Hamming Hausdorff distance(d_{31}) and

$$d_{31}((F,M),(G,N)) = \int_a^b w(x) \max j \; | \; F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj) \; | \; dx$$

a continuous fuzzy soft weighted Euclidean Hausdorff distance(d_{32}) respectively.

$$d_{32}((F,M),(G,N)) = \left[\int_a^b w(x) \max j \left(|F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2 \right) dx \right]^{1/2}$$

The generalized continuous fuzzy soft weighted distance(d_{30}) becomes a generalized continuous fuzzy soft normalized distance(d_{33}), if w(x)= $\frac{1}{(b-a)}$ for any $x \in [a,b]$, then(d_{33}):

$$d_{33}((F,M),(G,N)) = \left[\frac{1}{b-a}\int_a^b \max j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda} dx \right]^{1/\lambda}$$

Where $\lambda > 0$.

From the continuous fuzzy soft weighted Hamming Hausdorff distance(d_{31}),we get a continuous fuzzy soft normalized Hamming Hausdorff distance(d_{34}):

$$d_{34}((F,M),(G,N)) = \frac{1}{b-a}\int_a^b \max j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)| \; dx$$

And we have, the continuous fuzzy soft normalized Euclidean Hausdorff distance (d_{35}) from the continuous fuzzy soft weighted Euclidean Hausdorff distance (d_{32}):

$$d_{35}((F,M),(G,N)) = \left[\frac{1}{b-a} \int_a^b \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2 \, dx \right]^{1/2}$$

We derived a generalized hybrid continuous fuzzy soft weighted distance (d_{36}) from the generalized hybrid fuzzy soft weighted distance (d_{21}) as follows:

$$d_{36}((F,M),(G,N)) = \left[\int_a^b w(x) \left(\frac{1}{2li} \sum_{j=1}^{n} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda} + \frac{1}{2} \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda} \right) \right]^{1/\lambda}$$

where $\lambda > 0$

We get generalized hybrid continuous fuzzy soft normalized distance(d_{37}) from the generalized hybrid continuous fuzzy soft weighted distance(d_{36}). If $w(x)=\frac{1}{(b-a)}$ for all $x \in [a,b]$,then (d_{37}) is given as follows:

$$d_{37}((F,M),(G,N)) = \left[\frac{1}{b-a} \int_a^b \left(\frac{1}{2lx} \sum_{j=1}^{lx} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda} + \frac{1}{2} \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^{\lambda} \right) \right]^{1/\lambda}$$

Where $\lambda > 0$

From (d_{33}) we get a hybrid continuous fuzzy soft weighted Hamming distance (d_{38}) and a continuous hybrid continuous fuzzy soft weighted Euclidean distance (d_{39}). If λ=1, 2, then (d_{38}) and (d_{39}) are defined as follows

$$d_{38}((F,M),(G,N)) = \left[\int_a^b w(x) \left(\frac{1}{2li} \sum_{j=1}^{n} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)| + \frac{1}{2} \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)| \right) \right]$$

and

$$d_{39}((F,M),(G,N)) = \left[\int_a^b w(x) \left(\frac{1}{2li} \sum_{j=1}^{n} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2 + \frac{1}{2} \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2 \right) \right]^{1/2}$$

From (d_{38}) we get hybrid continuous fuzzy soft normalized Hamming distance(d_{40}) and hybrid continuous fuzzy soft normalized Euclidean distance(d_{41}).If $w(x)=\frac{1}{(b-a)}$ for all $x \in [a,b]$,then distance(d_{40}) and distance(d_{41}) are defined as follows:

$$d_{40}((F,M),(G,N)) = \left[\frac{1}{b-a} \int_a^b \left(\frac{1}{2lx} \sum_{j=1}^{lx} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)| + \frac{1}{2} \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)| \right) \right]$$

And a hybrid continuous fuzzy soft normalized Euclidean distance (d_{41}).

$$d_{41}((F,M),(G,N)) = \left[\frac{1}{b-a} \int_a^b \left(\frac{1}{2lx} \sum_{j=1}^{lx} |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2 + \frac{1}{2} \max_j |F(ei)_M^{\sigma(j)}(xj) - G(ei)_N^{\sigma(j)}(xj)|^2 \right) \right]^{1/2}$$

4. Application in decision making problem

In this section

Fuzzy Soft Sets distance similarity measures are useful to solve the decision making problem that contain uncertainty problems in the domains such as in social, economic system, pattern recognition, medical diagnosis and game theory coding theory. Now an example is illustrated for the decision making method based on Fuzzy Soft Sets distance similarity measures. The similarity measure of two Fuzzy Soft Sets (FSSs) can be applied to detect whether an ill person is suffering from a certain disease or not. An ill person having certain symptoms, who is suffering from typhoid,. First the Fuzzy Soft Sets (FSSs) is constructed for both illness and ill persons. Then we find the similarity measure of these two Fuzzy Soft Sets (FSSs). If they are significantly similar then we conclude that the person is possibly suffering from typhoid. The algorithm 1 of this method is as follows:

Algorithm ____

Step1. Construct a Fuzzy Soft Set (F, E) over the universe U based on an expert.
Step2. Construct a Fuzzy Soft Set (F_1, E) over the universe U based on symptoms.
Step3. Calculate the distances of (F, E) and (F_1, E).
Step4. Calculate similarity measure of (F, E) and (F_1, E).
Step5. Use similarity to estimate the results.

Example 3. Suppose that the universal set U contains only two elements 'yes' (typhoid) and 'no' (not typhoid) i.e. U ={ yes, no }. Here the set of parameters E is a set of certain visible symptoms. Let E = {e_1, e2, e3, e4, e5, e6}, where e_1 = loss of appetite, e2 = weight loss, e_3 = chest pain, e_4 = headache, e_5 = bone pain, and e_6 = wounds.
The tabular representations of Fuzzy Soft Sets are shown in Table 1, Table 2 and Table 3.

Table 1 Fuzzy soft set for (F, E) over U for typhoid as per
Medical expert opinion

(F_1,E)	e_1	e_2	e_3	e_4	e_5	e_6
yes	0.4	0.3	0	0.6	0.5	0.7
no	0.6	0.5	0.8	0.7	0.2	0.1

Table 2 Fuzzy soft set for (F_1,E) over U based on data of ill person

(F_2,E)	e_1	e_2	e_3	e_4	e_5	e_6
yes	0.7	0.5	0.9	0.2	0.6	0.4
no	0.2	0.3	0.3	0.8	0.1	0.5

Table 3 Fuzzy soft set for (F_2,E) over U based on data of ill person

(F_3,E)	e_1	e_2	e_3	e_4	e_5	e_6
yes	0.6	0.5	0.7	0.6	0.2	0.5
no	0.4	0.3	0.2	0.3	0.8	0.3

From Algorithm 1, we have Distances and Similarities between Fuzzy Soft Sets ((F,E),(F_1,E)) and Fuzzy Soft Sets ((F,E),(F_2,E)), which are shown in Table 2 and Table 3.

Table 4 Distances between ((F,E),(F_1,E)) and ((F,E),(F_2,E))

	d_i((F,E),(F_1,E))	d_i((F,E),(F_2,E))
d_2	0.308	0.291
d_3	0.108	0.107
d_4	0.3745	0.3321
d_5	0.410	0.380
d_6	0.200	0.100

Table 5 Similarities between ((F,E),(F_1,E)) and ((F,E),(F_2,E))

	$\tilde{\xi}_i$ ((F,E),(F_1,E))	$\tilde{\xi}_i$ ((F,E),(F_2,E))
$\tilde{\xi}_2$	0.692	0.709
$\tilde{\xi}_3$	0.892	0.893
$\tilde{\xi}_4$	0.6255	0.6679
$\tilde{\xi}_5$	0.59	0.62
$\tilde{\xi}_6$	0.8	0.90

The Table 5 shows that the similarities between two sets ((F,E) and (F_2,E)) of symptoms are maximum, therefore we conclude that the person is possibly suffering from typhoid.

5. Conclusion

The variety of new distance and similarity measures are proposed for fuzzy soft sets. The Fuzzy Soft distance and similarity measures can be applied in the variety of scientific fields such as decision making, pattern recognition, machine learning and market prediction, The different distance and similarity measures are based on Hamming distance, Euclidean distance, Hausdorff metric, normalized Hamming distance, normalized Euclidean distance, normalized Hausdorff distance, generalized Hamming distance, generalized Euclidean distance and generalized Hausdorff distance.

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